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3. The manuscript (an original ribbon copy and two duplicate copies) should be double spaced on one side of 8½-inch by 11-inch paper. Three copies of all illustrations, tables, etc., must be included.

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9. Technical papers intended for publication must be written in the third person.

10. The author should distinguish between a list of "Reading References" and a "Bibliography," which would encompass the subject of his paper.

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Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

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Note.--Part 2 of this Journal is the 1961-8 Newsletter of the Hydraulics Division.

DISCUSSION

Journal of the
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NEEDS IN SEDIMENTATION

By H. A. Einstein,[1] M. ASCE

SYNOPSIS

The answers to the question of which direction the research in a given field should go will depend, to a large degree, on the answerer's attitude. If he has the attitude of a pure scientist, he will direct the research towards the basic understanding of the processes, towards the analysis of the problem in general terms. If he is more of an engineer, he will first find out which phases of the problem are likely to become important in the future for the solution of the most pressing practical problems. He will then work on these phases, developing theories only as far as necessary for a most efficient solution of these problems. The engineering approach is adopted in this paper.

The most important changes to have taken place in the United States since about 1910 that have had an influence on the sediment motion in our rivers are ad follows: 1. During this period, the United States has essentially changed from a predominantly agricultural to a predominantly industrial economy. 2. During the same time, the total population and the water requirements per person have increased to such a degree that in almost all parts of the nation the naturally available local year-round water supply has become insufficient.

Both of these developments, the first one by its quest for energy and process water and the second directly by its increased water demand, have led to a large-scale development of all available fresh-water resources. The key to this development is the provision of storage space in which the available

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water can be stored during times of over-supply for consumption during the dry periods. Most of the artificial storage is provided in the form of surface reservoirs. These reservoirs change the regime of the rivers drastically, particularly with respect to their sediment characteristics:

1. Any large storage reservoir will permanently store practically the entire sediment load of the stream.
2. Part of the sediment storage occurs in the storage volume proper, part in the channel and valley bottom upstream by backwater effects.
3. The sediment load leaving the reservoir is usually nil or is restricted to very much reduced releases that have the purpose of flushing some of the sediment through the storage volume.
4. The hydrograph of flows available in the stream channel downstream from the reservoir is entirely changed and with it the annual sediment load carrying capacity.

Problems in and Above the Reservoirs.—Sediment deposition in the reservoir causes loss of storage volume; deposition above the reservoir causes back-water effects, swamping and flooding in the valley. These damages have hitherto been accepted as unavoidable and to a large degree as uncontrollable. Many examples of actual reservoirs and lakes have shown that the deposition of the sediment may follow many different patterns. Often, changes of patterns are clearly caused by particular influences, such as snags or vegetation, that suggest possible engineering control of the deposition process. If it is realized that both the storage volume in the reservoirs and the area of the extended valley bottoms above the reservoirs represent large and often irreplaceable values, it appears to be extremely wasteful to accept these losses, as they may occur, without question. A planned program of valley sedimentation, on the other hand, may not only reduce the loss of storage space in the reservoir but also create large valuable valley areas that are sufficiently high to prevent flood damage if used agriculturally or industrially. Such a program may also prevent the development of swamps with their health hazards.

What type of research is needed in connection with the development of such areas for sediment deposition? The first step is probably the systematic observation and description of the various deposition processes as they occur in existing prototype conditions, covering all materials from the bed load to the finest, for an extended range of river conditions. However, even such a very extended survey is only able to describe various chance combinations of the many variables that may or may not be important in their effect on the final result. Only a systematic study of the problem under the controlled conditions of a laboratory will permit the separation of the effects of the individual parameters.

Such laboratory studies, in turn, become effective only after the similarity conditions for deposition processes are known. Today, most model studies with movable bed deal with an approximately uniform bed material and try to find the geometric patterns of scour, transport, and deposition for that material under given flow conditions. The study of reservoir deposition would need the inclusion of all the fine components of the load in order to study the more complicated process of total load deposition.

After the development of methods operating with a large range of grain sizes under deposition conditions, and after scale effects for such processes are known, one can study specific questions.

Under which flow conditions is sediment deposited in a dense form; under which in a loose form? Is strength, or resistance against shear, in a sediment deposit related to density?

Under which flow conditions are sediment deposits segregated as to grain sizes and under which are mixtures deposited?

What are the relationships between range of grain sizes, density, and strength of various deposition patterns?

What are the agricultural values of the various types of deposits?

Can a systematic variation of the flow conditions achieve deposits that a continuous flow is not able to produce?

These are some of the most pertinent questions in connection with reservoir and valley deposits. Such questions may arise in connection with the engineering control of large-scale deposition processes and may be answered by systematic studies. It is certain that many more such problems will arise as soon as either deposition studies or deposition engineering in the prototype are undertaken.

The entire field of engineering control of deposition processes in and above reservoirs is practically unknown today. It is unquestionably possible to develop rules, methods and other tools for that purpose in the same way that similar know-how has been developed for the deposition of sediment in near-equilibrium streams for the purpose of channel stabilization.

There are mainly two reasons for the lack of such development. First, most of the reservoirs of significant size are located upstream of the area in which, at the time of construction, valley development was important. There is usually not enough damage expected in the reservoir proper for a generation or two to warrant the expenditure of funds and effort to prevent it. That means that the incentive is missing in cases in which new areas are developed. Secondly, where reservoirs are built into an already developed area, the local landowners above the reservoir do not benefit from it, but usually incur a loss of usuable land in various ways, and always feel that they are not sufficiently compensated for such losses. The operators of the reservoir, private or public, are then in the precarious position of having hostile neighbors in the entire area in which such controlled deposition may be practiced.

It is sometimes difficult for an individual to prove legally that he has been damaged by a very slowly progressing deposition process. In contrast, if engineering measures are undertaken to control deposition in the valley bottom, many local owners who may actually benefit from the operations, will go to court and claim damages. This is a rather costly undertaking for all parties involved. It can probably be avoided by organizing all landowners into a district that can then as a unit negotiate with the operator of the reservoir, after a concrete plan has been established for the operation of the deposition process. Unfortunately, sufficient general knowledge is not available today on these processes on which such a plan can be based. The economy of any such plan is usually even more in question. Only specific research in this field can provide the necessary background for such developments.

Sediment Problems Downstream from the Reservoirs.—The construction of a reservoir will have its greatest benefits in the river valley downstream through flood control and increased all-year water supply. The river valley downstream must be expected to develop economically at a much increased rate whether it remains agricultural or changes to urban and industrial development. The river that brings the fresh water from the reservoir will also

be called on to drain the valley bottom and, depending on its size, may even provide inexpensive transportation. Because the river channel is already provided. the laymen are apt to accept its shape and course as given and stable. When the majority of the high flows are held back by the reservoir, most or all of the flood danger appears to be eliminated.

To the engineer who is familiar with the management of alluvial rivers, the picture looks less rosy. He recognizes that the construction and operation of the reservoir severely reduces the sediment supply and radically changes the hydrograph of the river. These changes are so drastic that in most cases there is very little similarity between the new and the pre-reservoir river. Only the channel is the same. It is a sediment-lined channel that had usually reached at least near-equilibrium shape for the old river condition. It would be an extreme coincidence if the channel were still in equilibrium at the new flow and sediment conditions. The sediment supply is reduced. But also the channel capacity to transport sediment is somewhat reduced by the reduction of flood flows and the increase of medium flow durations. Usually the sediment supply is reduced more than the carrying capacity, causing the channel to erode. Often this development may be reversed in lower reaches after some sediment carrying tributaries have joined the main stream with the reduced sediment capacity. The immediate response of the river will be scour in the upper reaches. It will have acquired a full load when it arrives at the junction with any sediment carrying tributary. It may begin to aggrade at and below that junction. If the river channel is left free to develop at will, the upper reaches will, after a given time, adjust themselves to the new conditions. They will stabilize at a lower sediment rate, and the sediment load to the lower reaches will be reduced. It must be expected that during that process of adjustments the sediment supply to all parts of the river system will steadily change and call for continued readjustments.

After a channel section has finally stabilized its profile, for instance by coarsening its bed sediment, it is, unfortunately, not yet completely stable. The banks, that usually consist of a finer material than the bed, also begin to fail. They also seem to be stabilized only as long as finer material is deposited from the stream at about the same rate at which it scours. With the finer sizes missing in the load, the banks retreat steadily and the river channel widens. The flows begins to meander between banks and attack them with increased force where they impinge on them. It is not sufficient to stabilize the bed alone; the same must be done with the banks.

The valley bottom below a reservoir has been described as the area in which the maximum economic development must be expected to take place. The valuable ground will be in great demand and developments of all kind will soon begin to encroach on the river. A gradually enlarging river channel that is usually already too wide for the reduced flood flows is, therefore, a very undesirable solution.

Under these changed flow conditions, the engineer will ask again a number of questions when be tries to design a more desirable channel below a major reservoir.

What methods may be used to stabilize the profile of various streams downstream from reservoirs?

What methods may be used to stabilize the banks of various streams below reservoirs?

What cross section will be stable for such a changed stream?

What changes will occur in the composition of the stream bed due to the changed flow conditions?

Is it possible to stabilize channels for a range of sediment supply rates for each flow?

Is it more advantageous to stabilize the river channel before, immediately after, or any time after closure of the dam?

What types of structures are most effective for the stabilization of such channels and which materials should be used in their design?

Should stabilization begin at the dam and progress downstream or should it begin simultaneously over a considerable length of stream?

Should the changed stream after stabilization follow a meander pattern similar to the original stream or may it be straightened?

Many more such questions can and will be asked, particularly if special interests such as navigation, water quality control, and others are involved.

Now one must translate these engineering questions into research problems. This has already been attempted at the University of California, Berkeley, Calif. sediment laboratory for a number of years. Answers have been obtained from the resulting studies that, in many instances, point the way to the solutions of practical problems. First, the general conclusion was drawn from the previously stated arguments that any designed and stabilized channel must have artificially stabilized banks. The width between these banks is not automatically attained by the river, but must be chosen by the designer. This is in strong contrast to the alluvial river in its natural equilibrium state, in which the width reaches an equilibrium value depending on parameters of the flow, the sediment load, the vegetation and other factors.

The basic problem of the changed river as it arises in its most general form can then be stated as follows: Given the hydrograph and the sediment supply by amount and composition for all times, what slope, bed composition, depths and flow velocities will result as a long-range equilibrium for a given chosen channel width? Or the slope may be given and the channel width required. This basic question in its bold generality has not been solved, not even approximately. But a first step to its solution has been made at the University of California.

An equilibrium flow with a very heterogeneous load was disturbed by both an increase and then by a decrease of the sediment load. It was found that an increased load will behave quite rationally as may be expected from transportation laws for uniform sediment. The decreased load, on the other hand, resulted in a strong coarsening of the bed with progressively reduced scour. It was found that for each flow there exists a range of sediment sizes that move more slowly than the bulk and that all sizes above a limit practically do not move at all. These "slow-moving" and "non-moving" sizes appear to be the tools that nature uses to retard and even to prevent excessive downward scouring of channels. This process of bottom shingling and the resulting very limited bed scour has been observed in various rivers below large reservoirs. More work is needed for its quantitative prediction, particularly on the effect of flow variability.

In many rivers such non-moving sediment sizes are not contained in the river bed in sufficient amounts to limit down-scour to an acceptable depth. How can a similar protective layer of loose stone be introduced into the river bed artificially to limit down-scour? How does the necessary amount of rock depend on the rock size? These are all most important problems that can be solved experimentally and for which a complete answer does not exist.

In river reaches in which there is still an ample sediment supply in the bed sizes, but a lack of fines, the banks may need stabilization whereas the bed is still in equilibrium. The question arises in such rivers how deep the foundation of the banks must go. In any event must the toe of the bank be below the deepest instantaneous scour holes of the bed that may develop near the bank. This calls for a systematic and quantitative study of the shape of bed irregularities for the various flow and bed conditions. This extremely complex problem has recently found some interest in various quarters, but no theory is developed as yet by which quantitative river predictions can be made.

These are some of the most important developments that may be expected to occur in our rivers and an attempt has been made to predict the most important research problems that will arise with them. It is most interesting to remember that many of our notorious sedment carriers were clear and deep, quietly flowing, and tree lined streams at the time agriculture was brought to this land. They were changed into what they are by reckless methods of soil exploitation. It may now be necessary to change many such streams back to their original state because their waters have become sufficiently precious to conserve them.

CONCLUSION

The large storage reservoir is seen as the most important cause for new sediment problems is our rivers. Sediment transportation research should concentrate on the study of river conditions upstream and downstream from such reservoirs.

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COST ALLOCATION OF WATER PROJECTS IN CALIFORNIA

By Amalio Gomez,[1] F. ASCE

SYNOPSIS

This paper describes briefly the separable costs-remaining benefits method of allocating the cost of multiple-purpose water projects; summarizes the allocations made by the Corps of Engineers for several reservoir projects in the Central Valley of California; compares the results obtained; and makes some observations concerning the application of the method, based on actual experience.

INTRODUCTION

Cost allocation is as old as civilization itself. It probably began with the problem of sharing the cost of making war or of running the government. It may be said that cost allocation is a new name for describing a process quite similar to that of assessing taxes. It is now used primarily to describe the process of distributing the cost of a multiple-purpose water project among the functions that the project serves. It must be remembered that no fully satisfactory procedure has ever been devised for assessing taxes equitably in all cases. Similarly, no fully satisfactory procedure has ever been devised that will automatically yield equitable results in allocating the cost of a multiple-purpose water project, in all cases. The problem, therefore, concerns the computation of approximate values that reasonable men consider fair and just. Sound and experienced judgment is still one of the basic ingredients of any cost allocation.

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

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Experience in allocating the cost of multiple-purpose water projects goes back some 30 yr. Many methods have been devised and used with more or less satisfactory results. Some of the most common methods used include:[2]

Proportionate benefit method
Proportionate use method
Priority of use method
Alternative justifiable expenditure method
Separate projects method
Equal apportionment method
Incremental method
Direct cost method
Separable cost-remaining benefits method

METHOD

The method commonly in use by Federal agencies is the separable costs-remaining benefits method. The underlying principle of the method is that all

TABLE 1.—COST ALLOCATION

Item (1)	Flood control, in $1,000 (2)	Power, in $1,000 (3)	Irrigation, in $1,000 (4)	Navigation, in $1,000 (5)	Total, in $1,000 (6)
1. Benefits	500	1,500	350	100	2,450
2. Alternative cost	400	1,000	600	80	2,080
3. Benefits limited by alternative cost (lesser of items 1 and 2)	400	1,000	350	80	1,830
4. Separable costs	380	600	150	50	1,180
5. Remaining benefits (items 3–4)	20	400	200	30	650
6. Allocated residual cost	18	360	180	27	585
7. Total allocation (items 4 + 6)	398	960	330	77	1,765

project functions should share equitably in the savings to be realized by the multiple-purpose development. In the application of the method, project benefits for each function are first limited by the cheapest alternative cost of obtaining the same benefits. The method then assigns to each function all "separable cost," that is, the costs that were incurred by virtue of adding each function to the project. The remaining costs are called "joint costs," or "residual costs," and are distributed among the various project functions in proportion to the benefits remaining after deducting each separable cost from each project benefit. The method is best described in a report to the Inter-

[2] "Report on the Allocation of Costs of Federal Water Resources Development Projects," House Committee Print No. 23, 82nd Cong., 2nd Session, U.S. Govt. Printing Office, Washington 25, D. C.

Agency Committee on Water Resources.[3] This report was prepared by the Sub-Committee on Evaluation Standards, that developed the separable costs-remaining benefits method. A fair understanding of the method can be gained from the following quotation and Table 1 taken from the report:

> "The recommended method of cost allocation is illustrated below for a multi-purpose project for which the total project costs amount to $1,765,000. These include investment costs and operation, maintenance, and replacement costs, all reduced to a common time basis, and are expressed either as an average annual amount or a present worth amount."

APPLICATION

Present practice of the Corps of Engineers is to make preliminary cost allocations in connection with preauthorization reports. The preliminary cost allocation is brought up-to-date when funds for initiation of construction are being requested, and again as soon as practical after construction is initiated.

TABLE 2.—CONSTRUCTION STATUS

Reservoir Project (1)	Stream (2)	Status (3)
Pine Flat	Kings River	Completed
Isabella	Kern River	Completed
Folsom	American River	Completed
Success	Tule River	Under construction
Terminus	Kaweah River	Under construction
New Hogan	Calaveras River	Under construction
Black Butte	Stony Creek	Under construction

The final cost allocation is prepared just before the project is placed in operation. The practice followed during the last few years varied somewhat with each project, but, generally, it was to make a preliminary cost allocation before initiation of construction and fix the percentage distribution of project cost among its various functions at that time. Later, these percentages were applied to the actual construction costs.

PROJECT FUNCTIONS

Multiple-purpose reservoir projects built by the Corps of Engineers in the Central Valley of California serve two or more of the following functions: (a) navigation; (b) flood control; (c) irrigation; (d) municipal and industrial water supply; (e) hydroelectric power; (f) recreation; and (g) fish and wildlife. Cost allocations for Corps of Engineers projects are made by the Chief of Engineers, generally with the concurrence of the Bureau of Reclamation and other affected Federal, State, and local agencies. With some important ex-

3 "Proposed Practices for Economic Analysis of River Basin Projects," U. S. Govt. Printing Office, Washington 25, D. C., May, 1958.

ceptions, costs allocated to navigation and flood control are borne by the Federal Government. Costs allocated to irrigation are paid for by the water users, generally without interest, over a 40-yr period, under a contract with the Secretary of the Interior, made pursuant to the provisions of reclamation

TABLE 3.—SUMMARY OF ECONOMIC

Item (1)	Pine Flat, Kings River (2)	Isabella, Kern River (3)
		(a) Volumes, in
Reservoir capacity	1,000	570
Flood-control storage	1,000	540
Irrigation storage	1,000	535
Inactive storage	0	30
Average annual runoff	1,680	729
Average annual new water	165	47
Ave. ann. new and rereg. water	385	93
		(b) Economic Data,
First cost	35,760	21,093
Interest and amortization	1,390	744
Maintenance cost	174	118
Total annual cost	1,564	862
Flood-control benefits	3,377	1,353
Irrigation benefits	2,179	261
Power benefits	116	60
Total annual benefits	5,672	1,674
		(c) Allocated First
Flood control	19,250	15,469
Irrigation	14,250	4,573
Power	2,260	1,051
Total	35,760	21,093
		(d) Allocated Maintenance
Flood control	84	86
Irrigation	62	26
Power	28	6
Total	174	118

[a] Includes $590,000 for $59,000 acre-feet of municipal and industrial water supply.
[b] After deducting $2,140,000 creditable to generating, switching, and transmission facilities built by the U. S. Bureau of Reclamation, the cost of which is not included in the cost to be allocated.
[c] Includes $8,810,000 allocated to municipal and industrial water supply.
[d] Exclusive of cost of power facilities built by the Bureau of Reclamation.
[e] Includes $26,000 allocated to municipal and industrial water supply.

law, including the 160-acre limitation and other restrictions. Costs allocated to municipal and industrial water supply are repaid with interest, under a contract approved by the Secretary of the Army. Costs allocated to hydroelectric power are repaid with interest. The power is sold by the Secretary of the Interior.

Until recently, recreation and fish and wildlife have been considered as important, but secondary byproducts of the primary project functions. However, recreation and fish and wildlife are becoming more important every day in water resources planning. The Fish and Wildlife Coordination Act of

DATA AND ALLOCATED COSTS

Folsom, American River (4)	Success, Tule River (5)	Terminus, Kaweah River (6)	New Hogan, Calaveras River (7)	Black Butte, Stony Creek (8)
1,000 acre-feet				
1,000	80	150	325	160
400	75	142	165	150
912	75	142	310	150
88	5	8	15	10
2,746	137	415	157	428
1,024	7	18	38	57
1,024	26	55	72	57
in $1,000				
65,335	13,700	22,350	17,600	18,300
2,475	483	788	630	645
325	46	105	150	135
2,800	529	893	780	780
1,630	650	1,380	1,150	581
3,360[a]	55	147	314	508
360[b]	...	...	...	...
5,350	705	1,527	1,464	1,089
Cost, in $1,000				
16,765	12,404	19,190	11,238	11,000
42,180[c]	1,296	3,160	6,362	7,300
6,390[d]	...	...	...	...
65,335	13,700	22,350	17,600	18,300
Cost, in $1,000				
115	42	90	93	81
132[e]	4	15	57	54
78	...	...	...	...
325	46	105	150	135

1958 (P.L. 85-624) provided that in certain authorized projects and in all new projects, fish and wildlife will be considered for possible inclusion as a project function on the same basis as any other project function, and that the cost allocated thereto may be considered wholly or in part non-reimbursable. In the projects discussed here, no project costs have been allocated to recrea-

tion and fish and wildlife so far, but it may be that in some cases when the final cost allocations are made, some costs will be charged against these functions.

COST ALLOCATIONS MADE

Cost allocations have been made by the Corps of Engineers for seven multiple-purpose reservoir projects in the Central Valley of California. The names of these projects and the construction status are listed in Table 2.

The Pine Flat cost allocation was not based on any specific method of cost allocation, but gave weight to the results obtained by the use of several

TABLE 4.—COMPARISON OF COST ALLOCATION TO IRRIGATION.

Item	Project						
	Pine Flat	Isa-bella	Folsom	Success	Ter-minus	New Hogan	Black Butte
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Allocated first cost per acre-foot of average annual runoff, in dollars	8.48	6.27	12.15	9.46	7.61	40.52	17.06
Allocated first cost per acre-foot of new and re-regulated water, in dollars	37.01	49.17	34.58	49.85	57.45	88.36	128.07
Allocated first cost per acre-foot of reservoir storage usable for irrigation, in dollars	14.25	8.55	36.59	17.28	22.25	20.52	48.67
Annual cost to water users on basis of repaying capital cost in 40 equal payments without interest but including M. O. and R. charges:							
Per acre-foot of new water, in dollars	2.53	2.99	0.97	5.20	5.22	5.69	4.15
Per acre-foot of new and reregulated water, in dollars	1.09	1.51	0.97	1.40	1.71	3.00	4.15

methods. All other allocations have been made by the separable costs-remaining benefits method. Table 3 contains some descriptive data on the use of storage space as well as a summary of first and annual costs for each project. It also contains a summary of allocated first costs and maintenance, operation, and replacement costs. A further summary for each specific project with pertinent comments is given later.

It is extremely difficult to pass judgment on the fairness of a given allocation. For this reason, results obtained at one project are often compared with results obtained at other projects. Such comparisons can serve a useful purpose. However, they must be properly qualified because there are too many unrelated factors that affect the results. Among these factors are: (a) physical characteristics of the dam and reservoir site; (b) characteristics of the

stream such as runoff and extent to which already developed; (c) service area in which water is used; (d) developments in flood plain; and (e) the price level under which the project was constructed. With these qualifications in mind, the comparisons involving the cost allocated to irrigation given in Table 4, may be of some interest.

The preceding comparisons have been made on the basis of cost allocated to irrigation, because such costs are repayable and there is a tendency to judge the fairness of the allocation on the basis of repayable costs.

PINE FLAT DAM AND RESERVOIR

The cost allocation for the Pine Flat Project[4] was made in 1947. It was based on an estimated first cost of $35,760,000, of which $33,500,000 was for the dam and reservoir, and $2,260,000 was for penstocks for future power generation at the site. The allocation was as follows:

Flood control	$19,250,000
Irrigation	14,250,000
Power	2,260,000
Total	$35,760,000

The War Department Civil Appropriation Act, 1947, appropriated funds for the initiation of construction of this project subject to the proviso that:

> ". . . None of the appropriations for the Kings River and Tulare Lake project, California, shall be used . . . until the Secretary of War . . . with the concurrence of the Secretary of the Interior, shall have made a determination as to what the allocation shall be: provided further . . . that the agreement of concurrence shall be made not later than 9 months from the date of the enactment of this Act."

Because of the increase in price level and some design changes, when the dam was finished in 1954, the total cost was about $39,385,000, of which about $1,000,000 was for penstocks for future power. Because the allocation had been made at the specific request of Congress, and it had been furnished to and accepted by Congress, it could not be revised or done over again, on the basis of the actual construction cost, without a new directive from the Congress. This procedure has not been repeated with any of the subsequent projects.

The project is being operated for irrigation on the basis of interim contracts between the water users and the Bureau of Reclamation. A permanent contract is expected in the near future. The project has been eminently successful as illustrated by the following accomplishments during the 6-yr period since its completion in 1954:

Cash payments received from water users	$4,627,000
Cash payments received for storage of power water . .	$704,000
Estimated flood damages prevented	$20,000,000

ISABELLA DAM AND RESERVOIR

The cost allocation for Isabella Dam and Reservoir was made in 1955, when construction of the project was essentially completed. In this allocation,

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4 Printed in House Document 136, 80th Congress, 1st session U. S. Gov't. Printing Office, Washington 25, D. C.

annual cost and benefit data were capitalized on the basis of 2.5% Federal interest rate and a 50-yr amortization period, and the allocation was made on a "present worth" basis. Table 5 contains a summary of the allocation.

Even though no power generation at the site is involved in this project, the regulation provided by the project does benefit existing downstream power plants, and a cost allocation commensurate with the power benefits was made.

The project has been operated for irrigation since April, 1954 under terms of an interim contract between the water users and the Corps of Engineers.

TABLE 5.—SUMMARY OF THE COST ALLOCATION.

Item (1)	Flood control, in $1,000 (2)	Irrigation, in $1,000 (3)	Power, in $1,000 (4)	Total, in $1,000 (5)
Benefits	38,374	7,403	1,702	47,479
Alternative cost	24,445	22,942	1,702	49,089
Benefits limited by alternative costs	24,445	7,403	1,702	33,550
Separable costs	1,503	0	0	1,503
Remaining benefits	22,942	7,403	1,702	32,047
Allocated joint cost	16,424	5,300	1,218	22,942
Total allocation	17,927	5,300	1,218	24,445
Allocation of first cost	15,469	4,573	1,051	21,093

A permanent contract is being negotiated between the water users and the United States Bureau of Reclamation (USBR). Payments received so far for the interim irrigation service total $1,134,000, and flood damages prevented to date amount to $2,500,000.

FOLSOM DAM AND RESERVOIR

Folsom Dam was built by the Corps of Engineers. Folsom power plant and the Nimbus dam and power plant were built by the USBR. The entire project is being operated by the USBR as a part of the Central Valley Project. A preliminary cost allocation for Folsom Dam and Reservoir was made by the Corps of Engineers in 1953. The results of this allocation have been used by the USBR in various repayment analyses of the Central Valley Project. The preliminary cost allocation is given in Table 6. Note that the power benefits and the cost allocated to power are relatively small. The reason is that the power benefits were assigned to the power plants and to the dam and reservoir in such a manner as to provide for an equal ratio of benefits to cost for the power plants and for the dam and reservoir. Only the power benefits credited to the dam and reservoir are shown in Table 6.

An interesting sidelight in connection with this project is that no sooner had the dam been completed when the greatest flood of record, that of December, 1955 occurred, and the Sacramento metropolitan area was spared

a great catastrophe. It has been estimated that during the 1955 flood, the dam prevented about $20,000,000 in flood damages, that is more than the total cost allocated to flood control.

SUCCESS DAM AND RESERVOIR

A preliminary cost allocation for the Success Project was made in 1956. The percentage distribution of the project cost between flood control and irrigation was fixed at that time. Such percentages will be applied to the actual construction cost when the project is completed. A summary of the cost allocation is given in Table 7(a).

TABLE 6.—PRELIMINARY COST ALLOCATION, FOLSOM DAM AND RESERVOIR

Item (1)	Flood control, in $1,000 (2)	Irrigation, in $1,000 (3)	Municipal Water, in $1,000 (4)	Power, in $1,000 (5)	Total, in $1,000 (6)
Annual benefits	1,630	2,770	590	360	5,350
Alternative annual costs	1,610	2,160	590	360	4,720
Annual benefits limited by alternative costs	1,610	2,160	590	360	4,720
Separable annual costs	80	760	190	290	1,320
Remaining annual benefits	1,530	1,400	400	70	3,400
Allocated joint annual costs	670	610	170	30	1,480
Total annual costs	750	1,370	360	320	2,800
M. O. & R. costs	115	106	26	78	325
Interest and amortization	635	1,264	334	242	2,475
First costs	16,765	33,370	8,810	6,390	65,335

A contract is now being negotiated between the USBR and the water users for the amount to be repaid. The project is now under construction and is scheduled for completion in 1962.

TERMINUS DAM AND RESERVOIR

A preliminary cost allocation for Terminus Dam and Reservoir was made in 1958. The percentage distribution of the project cost between irrigation and flood control was fixed at that time. These percentages will be applied later to the actual construction cost. A summary of the actual allocation is given in Table 7(b).

A contract is being negotiated between the water users and the USBR for repayment of the part of the project cost allocated to irrigation. The project is under construction and is scheduled for completion in 1963.

NEW HOGAN DAM AND RESERVOIR

A preliminary allocation of cost for the New Hogan Dam and Reservoir was made in 1959. This allocation fixed the percentage distribution of the pro-

ject cost between flood control and irrigation, but the amounts will not be finally determined until the construction costs are known. A summary of the allocation is given in Table 7(c).

It is interesting to observe that construction funds were appropriated for fiscal year 1960, with the stipulation that construction not be initiated until a firm irrigation repayment contract had been executed. Such a contract between the State of California and the USBR was executed in March, 1960, and actual construction was started soon thereafter. This contract was entered into with the prime objective of permitting initiation of construction at an early date. The contract allows the USBR seven years to negotiate directly with the water users for the repayment of the amount allocated to irrigation. If the Bureau does not arrange for complete repayment in that period, the State of California will make the remainder of the payments due under the contract.

TABLE 7.—COST

Item	(a) Success Dam and Reservoir			(b) Terminus Dam	
	Flood control, in $1,000	Irrigation, in $1,000	Total, in $1,000	Flood control, in $1,000	Irrigation, in $1,000
Annual benefits	650	55	705	1,380	147
Alternative annual costs	529	529	1,058	893	442
Annual benefits limited by alternative costs	529	55	584	893	147
Separable annual costs	0	0	0	0	0
Remaining annual benefits	529	55	584	893	147
Allocated joint annual costs	479	50	529	767	126
Total M.O. and R. costs	42	4	46	90	15
Interest and amortization	437	46	483	677	111
First costs	12,404	1,296	13,700	19,190	3,160
Percentage of total costs	90.5%	9.5%	100%	85.9%	14.1%

BLACK BUTTE DAM AND RESERVOIR

The preliminary cost allocation for the Black Butte Dam and Reservoir was made in 1959. It fixed the percentage distribution of the project cost between flood control and water conservation. Water conservation includes irrigation and possibly municipal and industrial water supply. The fixed percentages will be applied later to the actual construction costs. A summary of the cost allocation is given in Table 7(d).

As in the case of the New Hogan Project, funds appropriated for fiscal year 1960 could not be utilized until a firm repayment contract was executed.

Such a contract was executed in March, 1960 by the USBR and the State of California. The contract is identical to the one just described for the New Hogan Project.

FUTURE PROJECTS

In addition to the projects described heretofore, a cost allocation has just been completed for the Oroville Project of the State of California. The purpose

ALLOCATIONS

and Reservoir	(c) New Hogan Dam and Reservoir			(d) Black Butte Dam and Reservoir		
Total, in $1,000	Flood control, in $1,000	Irrigation, in $1,000	Total, in $1,000	Flood control, in $1,000	Water conservation, in $1,000	Total, in $1,000
1,527	1,150	314	1,464	581	508	1,089
1,335	568	470	1,038	780	500	1,280
1,040	568	314	882	581	500	1,081
0	310	212	522	280	0	280
1,040	258	102	360	301	500	801
893	185	73	258	188	312	500
105	93	57	150	81	54	135
788	402	228	630	387	258	645
22,350	11,213	6,362	17,575	11,000	7,300	18,300
100%	63.8%	36.2%	100%	60.1%	39.9%	100%

of such allocation was to determine the justifiable amount of Federal contribution in the interest of flood control, such a contribution having been authorized by Congress subject to the amount being fixed by the Secretary of the Army and approved by the President. A preliminary cost allocation has also been made for the proposed Hidden Reservoir on Fresno River. The reports containing these allocations are now being considered by the Chief of Engineers. If approved, the results of the allocations will soon be made public.

Preliminary cost allocations are also being made in connection with prospective multiple-purpose storage projects on the Stanislaus, Tuolumne, Chowchilla, Merced, Mokelumne, and Yuba Rivers.

These possible future projects are noted simply to illustrate that the subject of cost allocation in the Central Valley of California is important and bears on many current and future projects.

APPRAISAL OF THE METHOD

A superficial examination of the separable costs-remaining benefits method may lead to the conclusion that there is finally available a specific and definite

method that will produce the same results no matter who is making the cost allocation. Far from it. The method assumes that a great deal of data are available that were gathered during the project formulation stage. Survey report investigations, that include project formulation and economic feasibility studies, are carried on generally with limited funds and often short cuts are needed. Therefore, it is seldom that all the data required for cost allocation purposes are available. Even the data that were originally available have become partly obsolete due to the time elapsed since they were collected and due to the change in economic conditions. The data most frequently deficient or unavailable are the cost of the cheapest alternative projects and the separable costs. The development of these data at reasonable cost requires the exercise of mature judgment. The most difficult part involves the determination of the magnitude and location of the physical works required to obtain the specified services in each case, with reasonable assurance that the combinations selected are the most economical. Normally, this is a time-consuming trial and error job. Hence, short cuts by experienced men are almost a practical necessity.

However, in spite of these drawbacks, the author believes that, as a general rule, the separable costs-remaining benefits method is by far superior to any other method with which he is familiar. It also has the advantage that it is beginning to be better understood and generally accepted. The allocations made by the Corps of Engineers in the Central Valley of California, by using this method, have been generally accepted as being fair and equitable by the water users and by the organizations involved.

ACKNOWLEDGMENTS

The cost allocations discussed herein have been made by Sacramento District of the Corps of Engineers over a 14-yr period. It is, therefore, difficult to give proper credit to the many fine military and civilian engineers that took part in the work, including: H. A. Morris, District Engineer; F. Kochis, Chief, Engineering Division; Wm. A. Doyle; and A. W. Zimmerman.

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VECTOR ASPECTS OF DYNAMIC SIMILARITY

By R. C. Kolf,[1] M. ASCE, and W. L. Reitmeyer[2]

SYNOPSIS

A study of the laws of dynamic similarity emphasizing the refinements imposed by the vector nature of the fluid flow variables is presented. As examples of the method proposed, the problems of turbomachinery and geometrically distorted models that have needed separate treatment in the past, because of their similarity distortions, are shown as logical extensions of the same theory.

INTRODUCTION

The bases for the prediction of the characteristics of flow in a fluid mechanics design problem are usually grouped in three general categories: (1) That of "engineering experience," (2) the laboratory method of studying each problem by means of scale models, and (3) the process of theoretical analysis. In most of the problems that face hydraulic engineers, the shape of the physical boundaries that confine the flow are complex and they are often in motion. Consequently, the prediction of the flow characteristics by purely analytical means is many times impossible because of the lack of presently available theoretical methods that are adaptable to such situations. The second method of prediction from the study of hydraulic models, utilizing whenever possible the best features of the other methods, has proved very useful in such situations.

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Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

THEORY OF DYNAMIC SIMILARITY

The laws of dynamic similarity are the criteria used in the design and analysis of hydraulic models. Depending on the type of model to be built and the characteristic to be studied, these criteria take on various degrees of sophistication. When only geometric similarity is of interest, the prediction equations can be written in terms of a single scale ratio, L_r, in which the subscripts m and p refer to model and prototype respectively.

$$\frac{L_m}{L_p} = L_r \qquad (1)$$

$$\frac{A_m}{A_p} = A_r = L_r^2 \qquad (2)$$

and

$$\frac{\overline{V}_m}{\overline{V}_p} = \overline{V}_r = L_r^3 \qquad (3)$$

In Eqs. 1, 2, and 3 $\overline{V}$ is the volume, L refers to the characteristic length, and A denotes the cross-sectional area.

A second class of variables involves time measurements as well as length measurements. For kinematic similarity a second scale ratio, namely the ratio of characteristic times, t_r, must be established.

$$V_r = \frac{V_m}{V_p} = \frac{L_r}{t_r} \qquad (4)$$

and

$$a_r = \frac{a_m}{a_p} = \frac{L_r}{t_r^2} \qquad (5)$$

in which V denotes the velocity vector and a is the acceleration.

A more complete listing of these prediction ratios is available in many standard reference works.[3] The significant characteristic to be noted at this point is that Eqs. 4 and 5 relate vector quantities. The division of one vector by another, as indicated by these vector ratios, has no known physical significance; hence the relationship so expressed is the ratio of the scalar or absolute magnitudes of the vectors. If this quantity is to have any pertinence in terms of similarity, it is necessary that each velocity or acceleration vector in the model have the same direction as the corresponding vector in the prototype. Then, in addition to Eqs. 4 and 5 being fulfilled, the flow patterns must be identical. This condition is ordinarily fulfilled, or at least approximated, by insisting on geometric similarity as a prerequisite for kinematic similarity because, for each boundary geometry, the ideal flow

[3] ASCE Manual of Engineering Practice No. 25, Hydraulic Models, 1942.

net is uniquely defined. Distortions of the flow net occur because of shear and must be accounted for by the further conditions of dynamic similarity.

Dynamic similarity requires that the ratios of all homologous forces in the two systems be the same. Limiting this study to cases in which only the pressure force, F_p, the viscous force, F_v, the gravity force, F_g, and the inertia force, F_i, are significant yields

$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_p)_m}{(F_p)_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_v)_m}{(F_v)_p} = F_r \quad \ldots\ldots\ldots \quad (6)$$

or, because the resultant force on a fluid element is called the inertia force

$$F_i = F_p \overset{\rightarrow}{+} F_g \overset{\rightarrow}{+} F_v \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

and

$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_p)_m \overset{\rightarrow}{+} (F_g)_m \overset{\rightarrow}{+} (F_v)_m}{(F_p)_p \overset{\rightarrow}{+} (F_g)_p \overset{\rightarrow}{+} (F_v)_p} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Again, insisting that the ratios indicated by Eqs. 6 and 8 are merely the ratios of the absolute magnitudes of the vectors involved, and that the homologous vectors on model and prototype must have the same directions in order to be similar, it can be seen that Eq. 6 is a limiting solution of Eq. 8. Other solutions of Eq. 8 are conceivable, but to define similarity of the resultant (inertia) force without specifying similarity of the component forces would result in a distortion of the geometry and the kinematics of the system in a manner that has thus far been undefined. There is no present utility of such a system.

The symbolic relationships for the forces can be developed as follows:

$$F_g = W = \gamma\,(\overline{V}) \propto \gamma\, L^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$F_p = \Delta p\, A \propto \Delta p\, L^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$F_v = \mu \frac{dV}{dy} A \propto \mu\, V\, L \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

and

$$\Sigma F = F_i = M\, a = \rho\, \overline{V}\, V \frac{dV}{ds} \propto \rho\, L^2\, V^2 \quad \ldots\ldots\ldots \quad (12)$$

in which W is the weight, γ refers to the specific weight, p indicates the pressure, μ denotes the dynamic viscosity, V is the velocity vector and ρ refers to the mass density. Eqs. 9, 10, 11, and 12 in the final form shown, all involve a proportionality constant. For instance, in the case of the gravity force, $F_g \propto \gamma\, L^3$. In order that the force ratios in Eq. 6, have the same value regardless of the particular force studied, these proportionality constants should be the same for the model as for the prototype. This will be the case

if the two are geometrically and kinematically similar. A hierarchy of similarity conditions is therefore as follows:

1. Geometric similarity may be obtained exclusive of the other classes.
2. In order to obtain complete kinematic similarity, geometric similarity must also be present, but in the ideal flow sense, dynamic similarity is not necessary.
3. For complete dynamic similarity, both geometric and kinematic similarity are ncessary.

The various dynamic similarity design criteria are fashioned by rearranging the terms in Eq. 6,

$$\frac{(F_i)_m}{(F_g)_m} = \frac{(F_i)_p}{(F_g)_p} = \frac{V^2}{L\,g} = \mathbf{F} \qquad (13)$$

$$\frac{(F_i)_m}{(F_v)_m} = \frac{(F_i)_p}{(F_v)_p} = \frac{V\,L\,\rho}{\mu} = \mathbf{R} \qquad (14)$$

and

$$\frac{(F_i)_m}{(F_p)_m} = \frac{(F_i)_p}{(F_p)_p} = \frac{\rho\,V^2}{\Delta p} = \mathbf{E} \qquad (15)$$

in which $\mathbf{F}$ is the Froude number, $\mathbf{R}$ refers to Reynold's number and $\mathbf{E}$ indicates Euler's number.

Other combinations are, of course, possible, but they would not be independent of those shown in Eqs. 13, 14 and 15 and are, therefore, unnecessary. If a geometrically similar model is fashioned according to one of these equations in such a way that it does not violate the other two, it will generally be dynamically similar to the prototype both in the sense of the absolute magnitudes of the variables and in the sense of the direction of the vector quantities. This is facilitated in many problems by the absence of one of these types of forces as a motivating factor, that obviates the necessity of conforming to all of the three equations (Eqs. 13, 14, and 15). Also, because the inertia force has been included in this study as the composite representation of all the other forces, one of the forces (and, therefore, one of the three equations) may be considered as being dependent on the others. Thus, a particular model design will generally necessitate observation of only one or possibly two of these ratios.

SPECIAL CASES

The vector conditions implied in the development of these laws of similarity limit the sole use of the Froude, Reynolds, and Euler numbers to problems in which the direction of the velocity and acceleration vectors is determined by the boundary geometry. In many problems this is not the case. Thus, designs that include varying degrees of curvilinearity, with or without the added complication of geometric distortion, have often been handled in the past as separate problems, and not as logical extensions of the same theory. The limits of the possible extension of the general laws have not been estab-

lished and, therefore, their effectiveness as tools of the hydraulic engineer has not been fully realized.

An example of such a problem that has been successfully handled by extending the laws of dynamic similarity is the vortex chamber. Essentially, this device consists of a circular or spiral shaped chamber with tangential and axial connections. It may be an enclosed (pressure force) system[4] as in the Thoma counterflow brake or an open system (gravity force) as in the Portland combined sewer diversion.[5] If only the Euler number and Reynolds number conditions must be satisfied, a particular value of the one parameter will be accompanied by the same value of the other in each system. The following equations then apply:

$$\frac{V^2}{g\,H} = \frac{\rho\,V^2}{\Delta p} = f(\mathbf{R}) \qquad (16)$$

$$Q = C\,A\sqrt{2\,g\,H} \qquad (17)$$

and

$$C = f'\,(\mathbf{R}) \qquad (18)$$

in which Q is the volume rate of flow, C denotes the coefficient of discharge and H refers to the head. In the vortex chamber problem, this analysis was insufficient because, for a particular boundary geometry, there exist an infinite number of possibilities for the direction of the velocity vectors depending on the strength of the vortex developed. This then is a case in which geometric similarity does not necessarily fix flow net similarity. In order to fix this similarity of vector directions Eq. 6 was amended to

$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_p)_m}{(F_p)_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_c)_m}{(F_c)_p} = F_r \qquad (19)$$

This adds another necessary group to $\mathbf{F}$, $\mathbf{R}$, and $\mathbf{E}$. Using the symbolic expression for centrifugal force (that is a component of the inertia force), it becomes

$$F_c = m\,\frac{u^2}{R} \propto \rho\overline{V}\,\frac{u^2}{R} \qquad (20)$$

and

$$\frac{(F_c)_m}{(F_i)_m} = \frac{(F_c)_p}{(F_i)_p} = \frac{u^2}{V^2} = \mathbf{K}^2 \qquad (21)$$

in which m is the mass, u denotes the tangential component of the velocity vector and $\mathbf{K}$ is the Kolf vortex number. This expression relates the normal component of the inertia force vector (and, therefore, also the acceleration vector) to the tangential component. Along with the geometric similarity

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4 "An Investigation of the Thoma Counterflow Brake," Transactions, ASME, Munich Hydr. Inst., 1953.

5 "Vortex Flow Through Horizontal Orifices," by J. C. Stevens and R. C. Kolf, Transactions, ASCE, Vol. 124, 1959.

conditions and the other dynamic similarity criteria **F**, **R**, and **E**, complete similarity is achieved. In this problem, Eqs. 16 and 18 are modified to

$$\frac{V^2}{g\,H} = \frac{\rho\,V^2}{\Delta p} = f\,(\mathbf{R},\mathbf{K}) \quad \text{(22)}$$

and

$$C = f'\,(\mathbf{R},\mathbf{K}) \quad \text{(23)}$$

Because viscous shear is significant only at low values of **R** (that normally do not occur in water flow problems), laboratory studies of the vortex chamber yield a curve establishing the dependence of the orifice coefficient, C, on **K**.[6]

HYDRAULIC MACHINERY

Another study that falls into this category is that of hydraulic machinery. Valid similarity laws have been developed for specific use in this field. However, these criteria (such as specific speed, unit speed, unit discharge) have not been represented as amplications of the previously described methods, but rather as substitutions for them, thus restricting the view of the dynamic similarity concept. The following presentation, on the other hand, follows directly from the basic theory presentation.

The pressure force and the viscous force should be considered in the case of a centrifugal pump, with the inertia force being again included in the study as the resultant of the two. If complete dynamic similarity is assumed, it should be possible to show the Euler number as a function of the Reynolds number, but this simplification of the problem would not prove useful. Because of the variability of the angular velocity of the impeller, kinematic similarity in the vector sense cannot be fixed by geometric similarity alone. The "speed ratio," $\frac{V}{u}$, that is the ratio of a characteristic velocity, V, to the tangential component of a characteristic velocity vector within the impeller, u, has been used to fix this kinematic similarity. This ratio is recognized to be the reciprocal of **K** from the vortex chamber consideration. It is, therefore, proportional to the square root of the ratio of inertia force to centrifugal force. Using the relationship $u \propto N\,D$ and $V \propto Q\,D^{-2}$

$$\mathbf{K}^{-1} \propto \frac{V}{u} \propto \frac{Q}{N\,D^3} \quad \text{(24)}$$

in which N is the angular velocity and D denotes the diameter. Most of the problems of engineering interest involve high Reynolds numbers. The pressure force is then the predominant force, and in the same way that Eq. 16 was modified to the form of Eq. 22 for the vortex chamber, the Euler number must be modified by Eq. 24 or an equivalent expression.

$$\frac{\rho\,V^2}{\Delta p} = f\left(\frac{Q}{N\,D^3}\right) \quad \text{(25)}$$

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6 "The Vortex Chamber as an Automatic Flow Control Device," by R. C. Kolf and P. B. Zielinski, Proceedings, ASCE, December, 1959.

and

$$\frac{H}{\frac{V^2}{2g}} = f'\left(\frac{Q}{N D^3}\right) \qquad (26)$$

Fig. 1 shows a plot of Eq. 26 for a vertical end-suction pump operated as a laboratory model, with an impeller diameter of 20-5/16 in., and a geometrically similar prototype with an impeller diameter of 50 in. The speed of the model was 1267.5 rpm and the prototype speed was 514 rpm. The characteristic velocity, V, in Eq. 26 was arbitrarily chosen as the average velocity in the exit pipe.

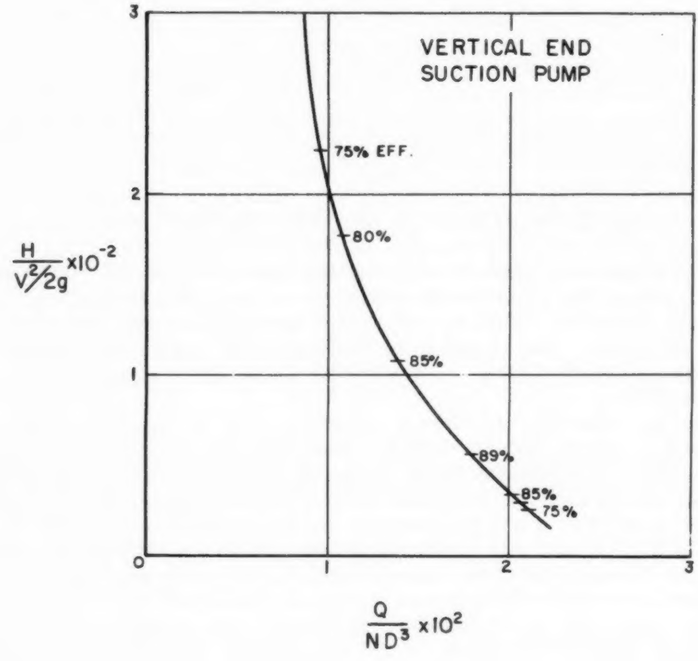

FIG. 1

To the scale shown it was not possible to differentiate the data for the two pumps, the "spread" being less than the thickness of the line. Because of the dynamic similarity defined by the curve, each point represents a particular efficiency for this design. Curves of this nature have also been used with the same success to represent the data for a single pump operated at various speeds.

From Eq. 20

$$F_c \propto \rho\ L^4 N^2 \qquad (27)$$

From Eq. 12

$$F_i \propto \rho\ L^4 t^{-2} \qquad (28)$$

It is of interest to note that

$$\rho_r\ L_r^4\ N_r^2 = \rho_r\ L_r^4\ t_r^{-2} \quad (29)$$

$$t_r = N_r^{-1} \quad (30)$$

$$Q_r = \frac{L_r^3}{t_r} = L_r^3\ N_r \quad (31)$$

$$V_r = \frac{L_r}{t_r} = L_r\ N_r \quad (32)$$

$$H_r = N_r^2\ L_r^2\ g_r^{-1} \quad (33)$$

and

$$P_r = N_r^3\ L_r^5\ \rho_r \quad (34)$$

It will be recognized that the Eqs. 29 through 34 describe the "laws of homologous operation" of turbo machines.

GEOMETRIC DISTORTION OF OPEN CHANNEL MODELS

The problems involved in applying the laws of similarity to the vortex chamber and to turbomachinery might be classified as resulting from a "kinematic distortion." That is, true vector similarity is not assured by geometric similarity. The situation encountered when geometric distortion is necessary, although falling in a separate category, has many of the same characteristics.

Geometric distortion is often a necessary evil in river hydraulics. The Reynolds and Froude criteria must be observed in this case. Although viscous shear is not a predominant force except at low values of Reynolds number, this restriction places a lower limit on the size of the model in order that the flow will be well into the turbulent range, and the modifications of the velocity profiles due to shear will be like those of the prototype. This lower limit on the size is, in many instances, unreasonable for economic reasons. When this is the case, geometrically distorted models are used in the hope that, although complete similarity is impossible, similarity with respect to an isolated phenomenon may be achieved. If the phenomenon to be studied involves a "movable bed," the conditions of sediment transportation must also be considered.

Hans Einstein, F. ASCE and Ning Chien[7] have given an orderly method for the design of such models. Several severe discrepançies have been noted, however, using these presently accepted techniques. This may be illustrated by considering the Froude number in which the characteristic length is considered to be the depth. The reason for this is that accelerations of the fluid are accompanied by changes in the elevation of the water surface. The justification is evidently an assumed two-dimensionality of the flow with the velocity variations in vertical, longitudinal planes more important to the problem

[7] "Similarity of Distorted River Models," by H. A. Einstein, N. Chien, Transactions, ASCE, Vol. 121, 1956.

than the transverse variations. This basic assumption can be shown to be a common source of error.

All ratios of homologous accelerations should have the same value if kinematic similarity is to be achieved.

$$a_r = g_r = 1 \qquad (35)$$

Let us suppose that the grain size of the moveable bed for the model is considered along with the velocity of the stream necessary to satisfy the Froude conditions based on depth, so that approximately similar sediment transport conditions are produced. The most common type of geometric distortion is to use a depth ratio, d_r, that is greater than the length ratio, L_r. Good engineering practice requires that the model and scales chosen be verified by reproducing known prototype developments with similar flows. Miscellaneous effects that have been observed in such model studies include[8] (1) excessive scour at bends and the beginning and end of obstructions such as deflecting dikes, (2) improper location of scour, (3) excessive bar building, and (4) formation of eddies in which none occurred in the prototype. All of these difficulties occur in regions of significantly curvilinear flow. The transverse velocity distribution in such regions (scour) is related to the normal force experienced by the fluid. The transverse currents (bar building) can also be linked to this phenomenon. If the model has been designed to give similarity of inertial conditions in the straight portions of the channel, the distorted normal acceleration will result in dissimilar conditions in the curved sections. Because the acceleration ratio is fixed, Eq. 35, another criteria that should be observed is

$$a_r = \frac{V_r^2}{R_r} = 1 \qquad (36)$$

Supposing that the depth ratio, d_r, is greater than the length ratio, L_r, by a factor of 6, that is not uncommon, and that the Froude condition has been observed, then

$$\frac{V_r^2}{d_r\, g_r} = 1 \qquad (37)$$

It should be noted that, because $R_r = L_r$, both of these equations cannot be satisfied simultaneously. Eq. 36 will have been violated by the same factor 6, resulting in severe distortion of the transverse conditions and greatly increasing the effects of the normal acceleration. Several possibilities of alleviating this condition exist, but there is no published record of their attempted use. One method would be to model the radius of curvature of severe bends to the depth ratio instead of the length ratio. This would amount to partially eliminating the distortion in the zones that normally cause difficulty. This type of distortion would introduce a psychological problem to the observer because it would not "look" like the prototype condition. It would not eliminate the need of the verification step.

[8] "Observed Effects of Geometric Distortion in Hydraulic Models," by K. D. Nichols, Transactions, ASCE, Vol. 104, 1939.

In rivers that normally flow at relatively large depths (small Froude numbers) it seems that the Froude number as computed from the depth loses its significance. In such channels, small accelerations of the fluid do not greatly affect the depth. Here then, it would seem better to evaluate the parameter using the length ratio, thus satisfying Eq. 36 in preference to Eq. 37.

CONCLUSIONS

1. The limitations imposed by the vector nature of the fluid flow variables necessitates reformulation of the statement of the laws of dynamic similarity in order to extend their utility into new realms of application. In some cases this extension may be accomplished by using the "Kolf vortex number" in addition to the classical similarity criteria.

2. A large class of problems in which variable amounts of "vorticity" are imposed by moving portions of the flow boundaries (such as pumps, turbines, and so on) may be included in the general study of similarity by adding this parameter (ratio of centrifugal to inertia force) that assumes like directions for homologous vectors.

3. Many of the discrepancies noted in geometrically distorted models arise from a violation of the normal acceleration requirements in zones of curvilinear flow. Recognition of this fact indicates possible model adjustments for compensation.

ACKNOWLEDGMENTS

The authors are grateful to the Marquette University Department of Theoretical and Applied Mechanics that, through the interests of W. G. Murphy, Chairman, gave full support to this study.

Financial support was given by the National Science Foundation, Engineering Sciences Program.

Data on geometrically similar pump operation was furnished by Fairbanks, Morse, and Co.

ADDITIONAL READING REFERENCE

1. Engineering Hydraulics, by H. Rouse, John Wiley and Sons, New York, 1950.

APPENDIX.—NOTATION

The following symbols adopted for use in this paper and for the guidance of discussers, conform essentially with "American Standard Letter Symbols for Hydraulics" (ASA Y10.2-1958) prepared by a committee of the American

Standards Association with Society representation, and approved by the Association in 1958:

a = acceleration

A = cross-sectional area

C = coefficient of discharge

d = depth

D = diameter

$\mathbf{E}$ = Euler number

F_c = centrifugal force

F_g = gravity force

F_i = inertia force

F_p = pressure force

F_v = viscous force

$\mathbf{F}$ = Froude number

g = acceleration due to gravity

H = head

$\mathbf{K}$ = Kolf vortex number

L = characteristic length

m = mass

N = angular velocity

p = pressure

P = power

Q = volume rate of flow

R = radius of curvature

$\mathbf{R}$ = Reynolds number

s = distance along streamline

t = time

u = tangential component of velocity vector

V = velocity vector

$\overline{V}$ = volume

W = weight

γ = specific weight

μ = dynamic viscosity

ρ = mass density

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THEORY OF WAVE AGITATION IN A HARBOR

By Bernard Le Méhauté[1]

SYNOPSIS

A first order theory for the value of the wave agitation in a simple basin caused by an incident periodical gravity wave is presented. This study is a natural result of experimental results and previous theory.

A harbor may be considered as a combination of discontinuities separated by a definite distance. The ultimate one, often in the form of a beach, is a total obstruction that may dissipate a part of the incident energy. The value of the agitation in such a basin is computed when the motion is predominantly two dimensional, as a function of the amplitude and period of the incident wave. A practical example applies the results of the theory directly.

The influence of the shape of the entrance, beach, wave traps, and so on on the value of the agitation are analyzed. The obtained formula for the magnitude of agitation emphasizes that the ratio of the length of the basin to the

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

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wave length is important because of multiple reflections. Resonance conditions are computed.

Results obtained by theory are compared with those obtained experimentally.

INTRODUCTION

This theory of agitation in a harbor could be considered as a natural outcome of the experimental results presented[2] in 1954 and[3] 1956 and of the author's theory of periodical gravity waves on a discontinuity.[4]

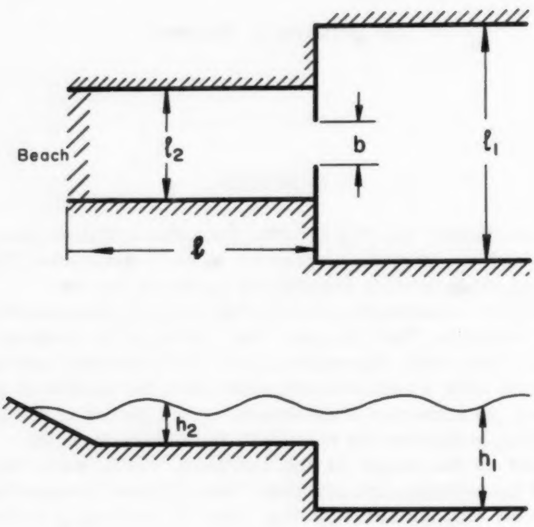

FIG. 1.—LAYOUT OF A BASIN

A harbor is considered herein as a combination of discontinuities separated by distances that define the lengths of the basins. The terminal discontinuity is, of course, a total obstruction. This total obstruction, frequently formed by a beach, dissipates a noticeable part of the incident wave energy.

With this assumption, the value of the agitation in a harbor may be computed from a general theory, that considers the behavior of waves acting on a combination of a number of discontinuities. This broad theory may be applied

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2 "Two Dimensional Seiche in a Basin Subjected to Incident Waves," by Bernard Le Méhauté, Proceedings, Coastal Engineering, 5th Conference, 1954.

3 "Mouvements de Résonance à Deux Dimensions dans une Enceinte sous l'action d'Ondes Incidentes," by F. Biesel and B. Le Mehaute, La Houille Blanche, July, August, 1956, No. 3, pp. 348–374.

4 "Periodical Wave on a Discontinuity," by B. Le Méhauté, Journal, ASCE, Vol. 86, No. 9, September, 1960.

to a number of such phenomena as waves passing over a shelf, agitation between two barriers and wave resonators, agitation in a succession of basins, and so on.

However, because of its importance in engineering practice, and because of the experimental data now at hand, only the case of a relatively simple basin, as shown by Fig. 1, is considered herein. It is, of course, a particular case in the more general theory of waves on a combination of discontinuities.

As in the author's previous work,[4] great use is made of the complex number method calculus.

It will be noticed that this method of complex number calculus provides a simple and powerful tool for the analysis of gravity wave motion. Solutions will be obtained by this method with a minimum of computation for cases which would usually be solved by finding a complicated potential function, ϕ. The method of the potential function, ϕ, is always tedious if there are discontinuities in the boundary conditions.

An example is presented which is consistent with the theory previously presented.

The theory is limited in that only terms of the first order of approximation are considered. That is, it deals only with the linear solutions in which, for example, the convective inertia effect, that is proportional to the square of the velocity, is neglected. This particular assumption seems valid for the majority of cases. Also, the theory deals only with the main vertical and horizontal components of sinusoidal motion. This stipulation tends to make the theory invalid as the discontinuity is approached, but it seems sensibly valid at distances of the order of two or three times the depth from the discontinuity. Three dimensional components, if existing, must have a limited effect. They are theoretically negligible when the width of the basin is less than half the length of the wave in the same domain. However, even for greater width, experiments indicate that this theory provides a fairly good approximation for the value of agitation, as long as no transverse resonance occurs. Transverse resonance is likely to occur when the width of the basin is very close to a multiple of half wave lengths.[4]

A linear periodical gravity wave may be defined by a rotating vector whose modulus is equal to half the wave height and whose argument is equal to the phase of the free surface at a given time and at a given point. In the same way a clapotis is defined by two vectors rotating in opposite directions.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

COMPLEX VALUE OF THE AGITATION IN A BASIN

Consider a basin aligned in the direction of the incident wave as shown by Fig. 2.

The incident wave, coming from the open sea arrives at the mouth of this basin with a half amplitude A_1 and a phase $\hat{A}_1$ at a given time.

This wave is considered to be defined by the complex number $\overline{A}_1$ of modulus $|\overline{A}_1| = A_1$, and argument $\hat{A}_1$. Similarly, the wave reflected by the over-

all layout (entrance and basin) is defined by a complex number $\overline{B}_1$ of modulus $|\overline{B}_1| = B_1$, and argument $\widehat{B}_1$. If there is no loss of energy at the discontinuity and inside the harbor

$$|\overline{A}_1| = |\overline{B}_1| \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The wave $\overline{A}_1$ is partially transmitted and partially reflected.

The transmitted wave is $\overline{\alpha}_1\ \overline{A}_1$ of half amplitude $\alpha_1\ A_1$ and phase $\left(\widehat{\alpha}_1 + \widehat{A}_1\right)$ and the reflected wave is $\overline{\beta}_1\ \overline{A}_1$ of semi-amplitude $\beta_1\ A_1$, and phase $\left(\widehat{\beta}_1 + \widehat{A}_1\right)$. The terms $\overline{\alpha}_1$ and $\overline{\beta}_1$ refer to the coefficients of transmission and reflection, respectively. They are defined by complex numbers of moduli α_1 and β_1 and arguments $\widehat{\alpha}_1$ and $\widehat{\beta}_1$, respectively.

These moduli $\left(\alpha_1 \text{ and } \beta_1\right)$ are equal to the ratios of the transmitted and reflected wave heights to the incident wave height, respectively. Similarly,

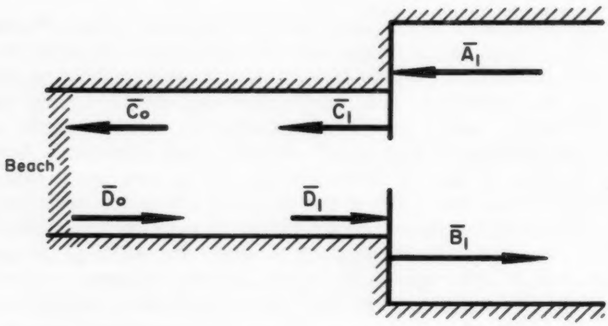


FIG. 2.—NOTATION OF PRIMARY WAVES

their arguments $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ are equal to the change of phases, caused by the discontinuity, of the transmitted and reflected waves, respectively.

Now, consider a wave defined by a complex number $\overline{D}_1$ arriving from the basin at the discontinuity. A part $\overline{\beta}_2\ \overline{D}_1$ is reflected within the basin, and another part $\overline{\alpha}_2\ \overline{D}_1$ is transmitted to the open sea. The terms $\overline{\alpha}_2$ and $\overline{\beta}_2$ are two complex numbers defining the transmission coefficient towards the open sea and the reflection coefficient towards the basin.

The four complex numbers $\overline{\alpha}_1$, $\overline{\beta}_1$, $\overline{\alpha}_2$, $\overline{\beta}_2$ or eight parameters moduli α_1, β_1, α_2, β_2 and arguments $\widehat{\alpha}_1$, $\widehat{\beta}_1$, $\widehat{\alpha}_2$, $\widehat{\beta}_2$ completely define the flow characteristics of the entrance of the basin.

Now, consider the four translatory primary waves $\overline{A}_1$, $\overline{B}_1$ in the open sea and $\overline{C}_1$, $\overline{D}_1$ inside the basin at the entrance to the harbor. Because all these waves have the same period

$$\overline{C}_1 = \overline{\alpha}_1\ \overline{A}_1 + \overline{\beta}_2\ \overline{D}_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2a)$$

and

$$B_1 = \overline{\alpha}_2\ \overline{D}_1 + \overline{\beta}_1\ \overline{A}_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2b)$$

The wave $\overline{C}_1$, moving inside the basin away from the entrance, arrives at the end of the basin, with a different phase $\hat{r}$. This phase is a function of the relative length of the basin with respect to the wave length in the basin

$$\hat{r} = -\frac{2\pi l}{L_2} \quad \ldots\ldots (3)$$

in which l is the length of the basin and L_2 the length of the wave in the basin.

The function $\hat{r}$ is negative because there is a negative phase difference between the wave at the entrance and the same wave arriving at the back. The wave height remains constant if friction is neglected. However, if friction is included, as produced for example by a wave trap or by lateral beaches, then the amplitude of the wave is reduced in the ratio $r(z < 1)$. As a first degree of approximation, r may be written

$$r = e^{-\frac{K l}{L_2}} \quad \ldots\ldots (4)$$

in which K is a friction coefficient.

Hence, the incident wave at the back of the basin becomes

$$\overline{C}_0 = \overline{r}\,\overline{C}_1 \quad \ldots\ldots (5)$$

in which $\overline{r}$ is a complex number of modulus r, and argument $\hat{r}$:

$$\overline{r} = r\,e^{i\hat{r}} \quad \ldots\ldots (6)$$

in which $i = \sqrt{-1}$.

In the case of a single basin, the obstruction is complete, and the wave $\overline{C}_0$ is reflected without change of phase. If $\overline{D}_0$ is this reflected wave, then

$$\hat{C}_0 = \hat{D}_0 \quad \ldots\ldots (7)$$

If reflection is assumed to occur without any loss of energy, then

$$C_0 = D_0 \quad \ldots\ldots (8a)$$

and

$$\overline{C}_0 = \overline{D}_0 \quad \ldots\ldots (8b)$$

However, the loss of energy, such as, that due to the breaking of a wave incident on a terminal beach, must often be considered. Hence, it is more accurate to write

$$\overline{D}_0 = p\,\overline{C}_0 \quad \ldots\ldots (9a)$$

and

$$D_0 = p\,C_0 \quad \ldots\ldots (9b)$$

in which p is a coefficient of reflection (≤ 1) that depends on the steepness of the incident wave, the slope, the roughness, perviousness, and the length of the beach. The coefficient p may be computed by use of theory[5] or be obtained experimentally as in the chart given by Messrs. Greslov and Mahe.[6] As a

5 "Etude du coefficient de réflexion d'une houle sur un obstacle constitue par un plan incliné," by Greslou and Mahé, Proceedings, Coastal Engineering, 5th Conference, 1954.

6 "Le pouvoir reflechissant des ouvrages maritimes exposés à l'action de la houle," by M. Miche, Annales des Ponts et Chaussées, Mai, Juin, 1951.

first approximation, p may be considered as a function of the ratio of the length of the beach to the width of the basin, assuming that all the wave energy arriving at a beach is dissipated in the breaking zone.

However for a long wave, such as a seiche type wave for which the period is of the order of 100 sec the reflection on a beach is almost total even when it has a very gentle slope so that we have Eq. 8b

$$\overline{C}_0 = \overline{D}_0 \quad \text{(8b)}$$

The wave $\overline{D}_0$ moving towards the entrance of the harbor becomes $\overline{D}_1$, such that

$$\overline{D}_0 = \overline{r}\,\overline{D}_1 \quad \text{(10)}$$

Eqs. 1, 2, 5, 9a, and 10 form a system of five equalities that permit the computation of the five unknowns $\overline{C}_1$, $\overline{C}_0$, $\overline{D}_0$, $\overline{D}_1$ and $\overline{B}_1$ as functions of the incident waves $\overline{A}_1$ and the coefficients $\overline{\alpha}_1$, $\overline{\alpha}_2$, $\overline{\beta}_1$, $(\overline{\beta}_2\ \overline{i})$ is each a complex number of modulus $|\ \overline{i}\ | = 1$ and argument $\hat{i} = 0$, that is the real number 1. (It should be noted that i is, in this case, different from the imaginary number $i = \sqrt{-1}$ used in Eq. 6.)

The following formulas are obtained from the previous equations:

$$\overline{C}_1 = \frac{\overline{\alpha}_1}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(11)}$$

$$\overline{C}_0 = \frac{\overline{r}\,\overline{\alpha}_1}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(12)}$$

$$\overline{D}_0 = \frac{p\,\overline{r}\,\overline{\alpha}_1}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(13)}$$

$$\overline{D}_1 = \frac{p\,\overline{r}^2\,\overline{\alpha}_1}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(14)}$$

and

$$\overline{B}_1 = \left(\overline{\beta}_1 + \frac{p\,\overline{r}^2\,\overline{\alpha}_1\,\overline{\alpha}_2}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\right)\overline{A}_1 \quad \text{(15)}$$

At the entrance the total value of the agitation in the basin is given by the equation

$$2\left(\overline{C}_1 + \overline{D}_1\right) = \frac{2\,\overline{\alpha}_1\left(\overline{i} + p\,\overline{r}^2\right)}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(16)}$$

and at the back by the equation

$$2\left(\overline{C}_0 + \overline{D}_0\right) = \frac{2\,\overline{r}\,\overline{\alpha}_1\,(1 + p)}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\,\overline{A}_1 \quad \text{(17)}$$

The value in the open sea, at the mouth, is

$$2\left(\overline{A}_1 + \overline{B}_1\right) = 2\left(\overline{i} + \overline{\beta}_1 + \frac{p\,\overline{r}^2\,\overline{\alpha}_1\,\overline{\alpha}_2}{\overline{i} - p\,\overline{r}^2\,\overline{\beta}_2}\right)\overline{A}_1 \quad \text{(18)}$$

The difference of phase between $\left(\bar{A}_1 + \bar{B}_1\right)$ and $\left(\bar{C}_1 + \bar{D}_1\right)$, that is argument $\dfrac{\bar{A}_1 + \bar{B}_1}{\bar{C}_1 + \bar{D}_1}$, gives the difference of phase of the agitation between the two sides of the entrance and enables an estimate to be made of the current at the entrance.

Verification.—A simple verification of these formulas consists in computing the absolute value of $\bar{B}_1$ when the loss of energy in the overall layout is zero, that is when $p = 1$ and $r = 1$. In this case, B_1 should be equal to A_1. This condition is obtained if

$$\left| \beta_1 + \frac{p\, \bar{r}^2\, \bar{\alpha}_1\, \bar{\alpha}_2}{\bar{1} - p\, \bar{r}^2\, \bar{\beta}_2} \right| = 1 \qquad (19)$$

or

$$\left(\bar{1} - \bar{\beta}_1\right)\left(\bar{1} - p\, \bar{r}^2\, \bar{\beta}_2\right) = p\, \bar{r}^2\, \bar{\alpha}_1\, \bar{\alpha}_2 \qquad (20)$$

Introducing Eqs. 19 and 20 and utilizing the relationships previously presented[4]

$$A\, \bar{\alpha}_1 + \bar{\beta}_2 = \bar{1} \qquad (21a)$$

and

$$\frac{\bar{\alpha}_2}{A} + \bar{\beta}_1 = \bar{1} \qquad (21b)$$

it is found that Eq. 20 is verified; that is $A_1 = B_1$.

Moreover the value of the agitation in the basin at the back of the harbor, when it is assumed that all the incident energy is destroyed by a terminal beach such that $p = 0$, is found to be

$$2 \mid \bar{C}_0 + \bar{D}_0 \mid = 2 \mid \bar{C}_0 \mid = 2\, r\, \alpha_1\, A_1 \qquad (22a)$$

At the entrance

$$2 \mid \bar{C}_1 + \bar{D}_1 \mid = 2 \mid \bar{C}_1 \mid = 2\, \alpha_1\, A_1 \qquad (22b)$$

that is also consistent, because these values represent the transmitted wave from the open sea into the basin.

ABSOLUTE VALUE OF THE AGITATION

The value of the agitation at the rear of the harbor is given by the absolute value $2 \mid \bar{C}_0 + \bar{D}_0 \mid$; at the entrance in the basin by the absolute value $2 \mid \bar{C}_1 + \bar{D} \mid$; and at the entrance in the open sea by the absolute value $2 \mid \bar{A}_1 + \bar{B}_1 \mid$. The amplification factor is given by dividing these values by the amplitude of the incident wave, A_1.

At the back of the harbor, the factor is easily found from Eq. 17 to be

$$\frac{A_g}{A_1} = \frac{2 \mid \bar{C}_0 + \bar{D}_0 \mid}{A_1} = \frac{2\, r\, (1 + p)\, \alpha_1}{\left[1 + \left(p\, r^2\, \beta_2\right)^2 - 2\, p\, r^2\, \beta_2 \cos\left(2\, \hat{r} + \hat{\beta}_2\right)\right]^{1/2}} \qquad (23)$$

From previous theory[4]

$$\beta_2 = \left(1 - A\, Z\, \alpha_1^2\right)^{1/2} \qquad (24)$$

and

$$\cos \hat{\beta}_2 = \frac{2 - \alpha_1^2 (1 + Z)}{2\left(1 - A\ Z\ \alpha_1^2\right)^{1/2}} \qquad (25)$$

in which

$$Z = \frac{L_2\ l_2}{L_1\ l_1} \qquad (26)$$

The terms l_2 and l_1 define the widths of the domains, 2, the open sea, and 1, the basin, respectively and L_2 and L_1 are the wave lengths in these two domains. The term L is a function of the period T and the depth h:

$$L = \frac{g\ T^2}{2\ \pi} \tanh \frac{2\ \pi\ h}{L} \qquad (27)$$

In addition $\left(m = \frac{2\ \pi}{L}\right)$

$$A = \frac{1 + \dfrac{2\ m_2\ h_2}{\sinh\ 2\ m_2\ h_2}}{1 + \dfrac{2\ m_1\ h_1}{\sinh\ 2\ m_1\ h_1}} \qquad (28)$$

Hence, it is possible to express the value of the agitation as a function of Z, A, and α_1.

The term α_1 may be expressed as a function of the shape of the discontinuity by the empirical formula

$$\alpha_1 = \left(\frac{b}{l_2}\right)^{1/2} \left(\frac{l_1}{l_2}\right)^{1/4} \frac{2}{1 + A\ \dfrac{L_2}{L_1}} \qquad (29)$$

in which b is the width of the entrance of the basin.

Finally, Eqs. 3, 23, 24, 25, 26, 28 and 29 permit the computation of the value of the agitation at the rear of the basin for a given incident wave of amplitude 2 A_1 and period T.

PRACTICAL FORMULAS

The value of the agitation at the back of the basin is given by the theoretical formulas

$$A_g = \frac{2\ r\ (1 + p)\ \alpha_1\ A_1}{\left[1 + \left(p\ r^2\ \beta_2\right)^2 - 2\ p\ r^2\ \beta_2 \cos\left(2\ \hat{r} + \hat{\beta}_2\right)\right]^{1/2}} \qquad (23)$$

in which r is the damping coefficient of the amplitude of the wave travelling from the entrance to the back of the basin; p is the coefficient of reflexion of the back of the harbor (p is small when this back is a beach); α_1 is the coefficient of transmission for the wave arriving from the open sea into the basin, and is computed from Eq. 29; β_2 is the coefficient of reflexion of the entrance for the wave travelling inside the basin in the direction of the open sea. β_2 is given as a function of α_1 by Eq. 24; $\hat{r}$ is the difference in phase in the basin between a wave at the entrance and a wave at the back of the harbor. $\hat{r}$ is

given by Eq. 3; $\hat{\beta}_2$ is the difference in phase between the wave reflected by the entrance inside the basin in the direction of the basin and the wave travelling in the direction of the open sea also inside the basin; $\hat{\beta}_2$ is zero in the case of a total obstruction and equal to π in case of a total opening in a wide and deep open sea; and $\hat{\beta}_2$ is given by Eq. 25.

In these formulas, use is made of coefficients: the shoaling coefficient $A(m = 2\pi/L)$ as defined by Eq. 28 and the change of domain coefficient Z caused by any change from the open sea to the basin, and defined by Eq. 26.

Example.—Consider a basin as shown by Fig. 2 in which the length of the basin $l = 1450$ ft; the width of the basin $l_2 = 500$ ft; the length of the mouth $b = 187$ ft; and the depth of the basin $h_2 = 32$ ft. A terminal beach exists with a very gentle slope and a length of 340 ft. The open sea (or another basin) is 66 ft deep (h_1) and 625 ft wide (l_1). The incident wave has a period of 10 sec and a height of $2\,A_1 = 15$ ft.

Preliminary Computation.—A 10 sec period wave in the basin in a depth $h_2 = 32$ ft, has a wave-length $L_2 = 300$ ft. A 10 sec period wave in the open sea in a depth $h_1 = 66$ ft has a length: $L_1 = 400$ ft. Hence

$$\frac{L_2}{L_1} = 0.75$$

Because $l_2 = 375 \text{ ft} > \frac{L_2}{2}$, and $l_1 = 750 \text{ ft} > \frac{L_1}{2}$ the theory is not strictly applicable but should provide a fairly good approximation to the value of the agitation. A transverse resonance is unlikely because l_2 and l_1 are not close to any multiple of $\frac{L_2}{2}$ and $\frac{L_1}{2}$, respectively.

Computation of the shoaling factor A by means of Eq. 28 proceeds as follows:

$$2\,m_2\,h_2 = 2\,\frac{2\,\pi}{L_2}\,h_2 = \frac{(4\,\pi)\,(32)}{300} = 1.34$$

$$\sinh 2\,m_2\,h_2 = 1.78$$

$$2\,m_1\,h_1 = 2\,\frac{2\,\pi}{L_1}\,h_1 = \frac{(4\,\pi)\,(66)}{400} = 2.07$$

and

$$\sinh 2\,m_1\,h_1 = 3.90$$

Substituting into Eq. 28 yields

$$A = \frac{1 + \dfrac{1.34}{1.78}}{1 + \dfrac{2.07}{3.90}} = 1.14$$

Substituting

$$\frac{l_2}{l_1} = \frac{500}{625} = 0.80$$

into Eq. 26 yield the change of domain coefficient

$$Z = 0.75 \times 0.80 = 0.60$$

Deriving the values

$$\left(\frac{l_1}{l_2}\right)^{1/4} = (1.25)^{1/4} = 1.06$$

$$\left(\frac{b}{l_2}\right)^{1/2} = \left(\frac{187}{500}\right)^{1/2} = 0.61$$

and

$$\frac{2}{1 + A\,\dfrac{L_2}{L_1}} = \frac{2}{1 + 1.14 \times 0.75} = 1.08$$

and substituting into Eq. 29 yields the transmission coefficient

$$\alpha_1 = 1.06 \times 0.61 \times 1.08 = 0.70$$

From Eq. 24 the reflection coefficient is found to be

$$\beta_2 = \left[1 - 1.14 \times 0.60 \times (0.70)^2\right]^{1/2} = 0.82$$

Substituting into Eq. 25

$$\cos \hat{\beta}_2 = \frac{2 - (0.70)^2\,(1.14 + 0.60)}{2 \times 0.82} = 0.70$$

the phase of the entrance is found to be

$$\hat{\beta}_2 = 45^\circ$$

From Eq. 3 the phase lag of the basin is found to be

$$2\,\hat{r} = -\,2\,\frac{2\,\pi}{L_2}\,l = -\,10 \times 2\,\pi + 120^\circ$$

Hence,

$$\cos\left(2\,\hat{r} + \hat{\beta}_2\right) = \cos\,(120^\circ + 45^\circ) = -\,0.97$$

It is assumed that the friction effect inside the basin is given by putting $r = 0.95$.

The terminal beach reflection coefficient p is then computed. When the beach length λ is small relative to the width of the basin l_2, the coefficient of reflection of the back of this basin can be computed by assuming that the wave energy dissipated is proportional to the beach length so that:

$$p = \left(\frac{l_2 - \lambda}{l_2}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

When λ is of the same order as l_2, it is more accurate to compute the reflection coefficient as a function of the beach length, its slope and the steepness of the incident wave $\frac{2\,C_0}{L_2}$. In this example, it is assumed that $p = 0.1$.

Now the value of the amplification factor may be computed by the use of the theoretical Eq. 23:

$$\frac{A_g}{A_1} = \frac{0.95 \times (1 + 0.1) \times 0.70}{\left[1 + (0.10 \times 0.90 \times 0.82)^2 - 2(0.10 \times 0.90 \times 0.82)\,(-\,0.97)\right]^{1/2}} = 1.24$$

and the value of the amplitude of the agitation at the back of the harbor is

$$A_g = 0.62 \times 2\,A_1 = 0.62 \times 15 = 9.3 \text{ ft}$$

It is likely that this value is slightly over-estimated. Because $l_2 > \frac{L_2}{2}$ and $l_1 > \frac{L_1}{2}$, it is apparent that some secondary three dimensional motions are superimposed on the primary two dimensional motion.

The Importance of the Ratio of the Length of the Basin to the Wave Length.— It should be noted that a slight variation in the term $(2\hat{r})$ in the expression $2\,p\,r^2\,\beta_2 \cos\left(2\hat{r} + \hat{\beta}_2\right)$ usually has a large effect on the value of the agitation.

If l is of the same order as L_2, a slight variation of T, does not change the value of the cosine given by the previous formula nor, consequently, the agitation very much. In practice, this is the case for long waves such as tides in a bay or seiches in a basin. But as far as ordinary wind generated waves and swell are concerned, l is generally much greater than L_2, and, a very small change of the period of the incident wave T, causes a change in the value of $\hat{r}$ that introduces a great variation in the value of $\cos\left(2\hat{r} + \hat{\beta}_2\right)$ and consequently in the value of the agitation.

Because waves in the field are always irregular it is necessary to consider the "degree of agitation" that could be found by computing the mean value of the agitation for a complete cycle of $\left(2\hat{r} + \hat{\beta}_2\right)$, or more simply of $2\hat{r}$ when the wave length, that is the wave period, varies.

It is interesting to note that the "degree of agitation" does not depend on $\hat{\beta}_2$, whereas the value of the agitation for a given period is very much influenced by the value of $\hat{\beta}_2$.

RESONANCE

Maximum agitation occurs when:

$$|\,\bar{1} - p\,\bar{r}^2\,\bar{\beta}_2\,| = 1 - p\,r^2\,\beta_2 \quad \ldots\ldots\ldots\ldots (31)$$

that is

$$2\hat{r} + \hat{\beta}_2 = 0\ (+\,2\,n\,\pi) \quad \ldots\ldots\ldots\ldots (32)$$

that gives

$$l = n\,\frac{L_2}{2} + \frac{\beta_2}{2\,\pi}\,\frac{L_2}{2} \quad \ldots\ldots\ldots\ldots (33a)$$

or

$$L_2 = l\,\frac{4\,\pi}{2\,n\,\pi + \hat{\beta}_2} \quad \ldots\ldots\ldots\ldots (33b)$$

If the water is shallow such that $L = T\sqrt{g\,h}$, then the periods of resonance are given by

$$T = \frac{l}{\sqrt{g\,h_2}}\,\frac{4\,\pi}{2\,n\,\pi + \hat{\beta}_2} \quad \ldots\ldots\ldots\ldots (34)$$

The corresponding maximum amplitude of agitation is equal to

$$Ag_{max} = \frac{2\,r\,(1 + p)\,\alpha_1}{1 - p\,r^2\,\beta_2}\,A_1 \quad \ldots\ldots\ldots\ldots (35)$$

The effect of friction is apparent where the values of p and r both tend to reduce the value of the amplitude of resonance, as would be expected intuitively.

Minimum agitation occurs when

$$\mid \bar{1} - p\,\bar{r}^2\,\bar{\beta}_2 \mid = 1 + p\,r^2\,\beta_2 \quad \ldots\ldots\ldots\ldots (36)$$

That is, when

$$2\,\hat{r} + \hat{\beta}_2 = \pi\,(+\,2\,n\,\pi) \quad \ldots\ldots\ldots\ldots (37)$$

or

$$l = n\,\frac{L_2}{2} + \frac{L_2}{4} + \frac{\beta_2}{2\,\pi} \times \frac{L_2}{2} \quad \ldots\ldots\ldots\ldots (38a)$$

and

$$L_2 = l\,\frac{4\,\pi}{2\,n\,\pi + \pi + \hat{\beta}_2} \quad \ldots\ldots\ldots\ldots (38b)$$

The minimum amplitude of agitation is equal to

$$A_{g_{min}} = \frac{2\,r\,(1 + p)\,\alpha_1}{1 + p\,r^2\,\beta_2}\,A_1 \quad \ldots\ldots\ldots\ldots (39)$$

EXPERIMENTAL VERIFICATION

The theory presented herein has been verified by a number of experiments conducted in the Sogreah Laboratory at Grenoble, France. Details of this experimental work have been presented previously.[2,3] For each case experimentally analyzed, the following results were obtained:

1. The curves of the coefficient of amplification $\frac{A_g}{A_1}$ are given as functions of the relative length of the basin for various shapes of the entrance and various characteristics of domain 1, the open sea, and domain 2, the basin. That is they are given as functions of the opening of the basin and the depths and widths of these two domains. The theoretical curves are computed assuming no loss of energy in the basin, that is

No terminal beach; $p = 1$

No damping of the wave: $r = 1$

These theoretical curves are compared with the experimental results in Figs. 3, 4, 6, and 7.

2. The maximum value of the agitation that occurs, of course, at resonance has been analyzed and presented in Fig. 5 for the case of a basin closed by a simple obstruction. The computed values presented assume
a) no friction
and b) a constant value of the friction terms p and r.

These two theoretical curves are compared with the experiment.

Successive computations have been made for the following harbor entrances:

1. Obstruction in a flume. (The domains 1 and 2 have the same characteristic.) (Figs. 3, 4 and 5)
2. Change of width, constant depth, and full opening. (Fig. 6)
3. Change of depth, constant width, and full opening. (Fig. 7)

It has also been verified that this theory could be applied to a number of other combinations with the same degree of accuracy.

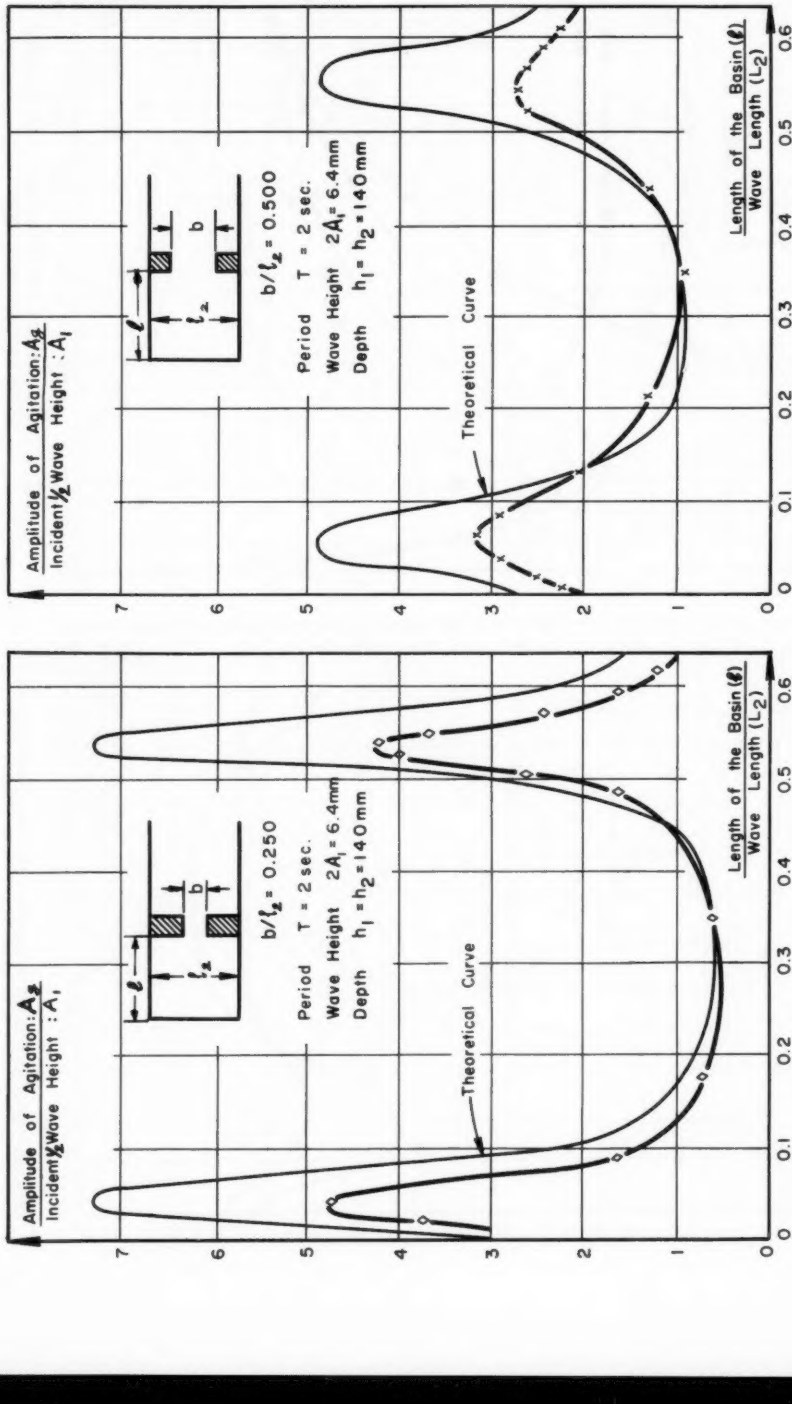


FIG. 3.—AGITATION IN A BASIN WITH AN OBSTRUCTION

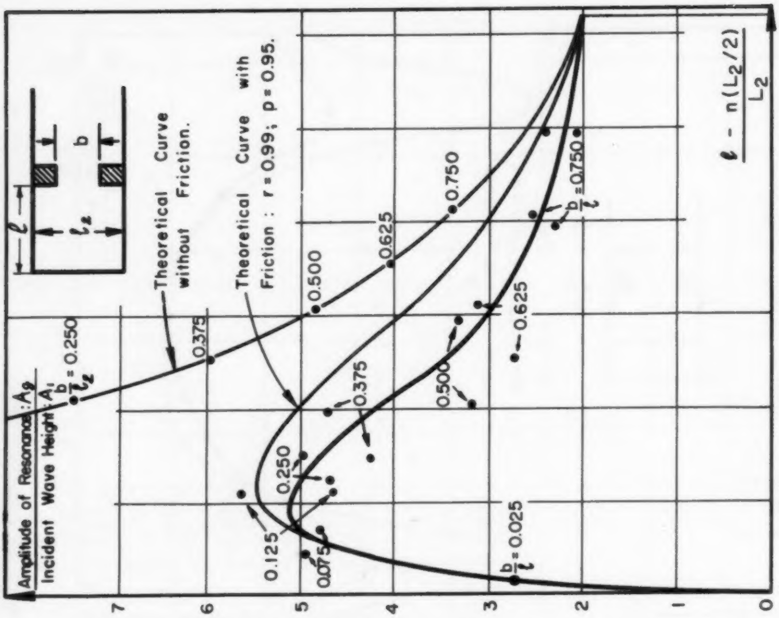


FIG. 5.—AMPLITUDE OF RESONANCE IN A BASIN WITH AN OBSTRUCTION

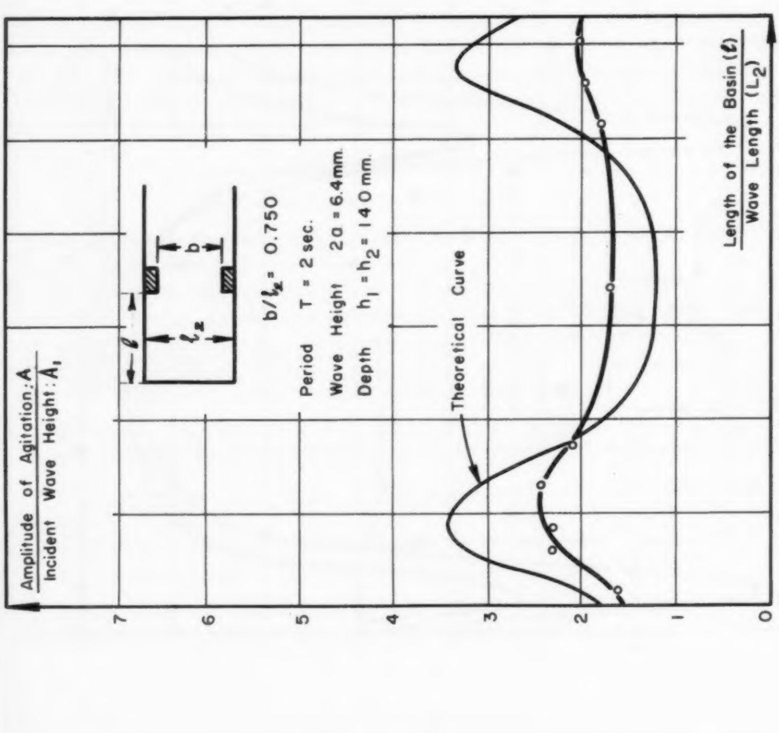
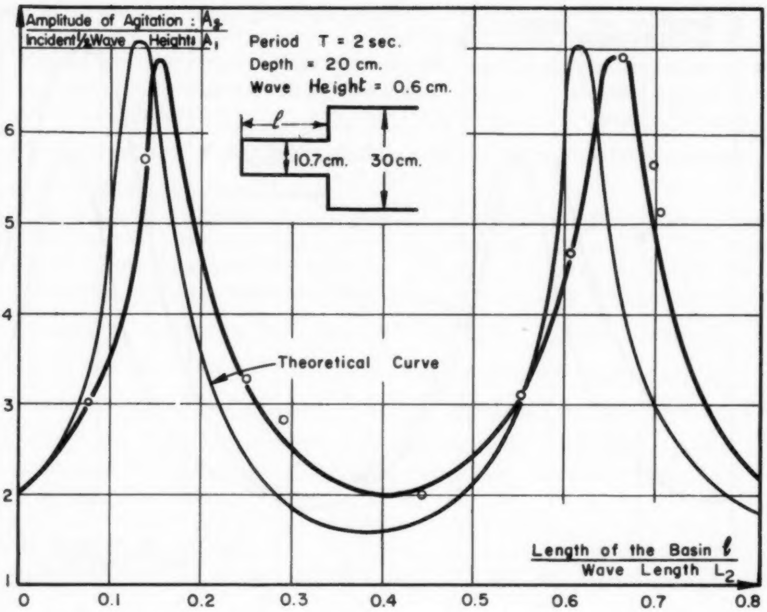


FIG. 4.—AGITATION IN A BASIN WITH AN OBSTRUCTION



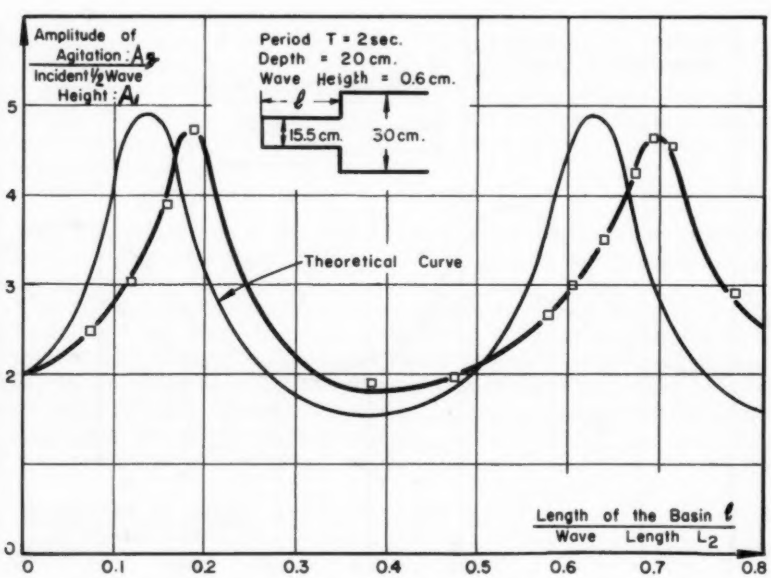


FIG. 6.—AGITATION IN A BASIN WITH A CHANGE OF WIDTH

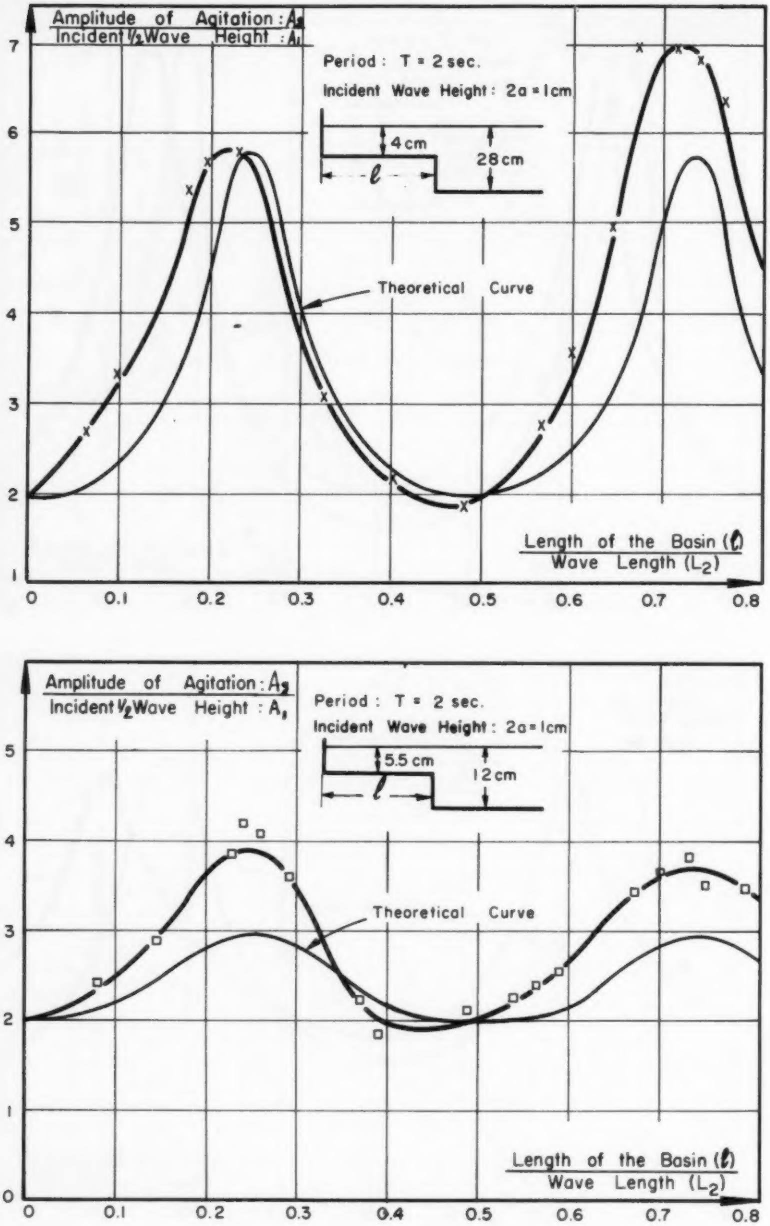


FIG. 7.—AGITATION IN A BASIN WITH A CHANGE OF DEPTH

The experimental and theoretical data presented in Figs. 3, 4, 5, and 6 are considered to exhibit fairly good agreement.

In the case of a basin closed by an obstruction, the friction has, as would seem logical, an important effect particularly on the maximum value of the agitation that occurs at resonance. Here it is necessary to take into account the friction coefficients r and p, however their values are usually difficult to determine accurately. The theory, particularly concerning the values of $\frac{b}{l_2}$ causing the resonance, is well verified.

For the case of a simple change of width, the friction effect may be neglected, ($p = 1$, $r = 1$) except when a terminal beach exists: $p < 1$. The computed values of the maximum (resonance) agitation is particularly well verified by the experiments, but there is a slight discrepancy between the theoretical values of $\hat{\beta}_2$ causing the resonance and those obtained by experiment. This means that the value of cos $\hat{\beta}_2$ calculated by introducing $\alpha_1 = (l_1/l_2)^{1/4}$ is incorrect as far as the value of $\hat{\beta}_2$ is concerned. As previously seen, the degree of agitation caused by an irregular wave does not depend on $\hat{\beta}_2$. Hence, the computation of the degree of agitation caused by an irregular wave is not affected by this discrepancy.

In the case of a simple change of depth, the occurrence of resonance as a function of l/L_2 is well verified. However, the magnitude of this maximum obtained experimentally seems greater than that computed from the theory.

SUMMARY

The results presented and cited indicate that the theory is generally well verified by experiments and can be successfully applied to compute the period and the amplitude of resonance, particularly if friction effects are small. Above all, the degree of selectivity of resonance given by the theory is always in accordance with the experimental results. This is shown by the relative thickness of the peaks of the agitation curves.

The following is found both by theory and experiment:

1. That the occurrence of resonance increases with a change of depth and a change of width, and decreases with an obstruction such that it tends to zero when this obstruction tends to be total;

2. That the value of the agitation at resonance increases as l_1/l_2, L_1/L_2 or h_1/h_2 and l_2/b increase. It will be noticed that the value of agitation tends to infinity with l_2/b, that is in the case of total obstruction, when the friction is neglected, but at the same time the occurrence of resonance tends to zero. However, when friction effects are taken into account, the value of the agitation tends to zero when l_2/b tends to infinity.

3. That resonance occurs when

$$n \frac{L_2}{2} < l < \left(n + \frac{1}{2}\right) \frac{L_2}{2}$$

for all cases;

$$l = n \frac{L_2}{2}$$

in the case of a complete obstruction;

$$l = \left(n + \frac{1}{2}\right)\frac{L_2}{2}$$

in the case of a change of depth without obstruction;

4. That the minimum agitation occurs when:

$$\left(n + \frac{1}{2}\right)\frac{L_2}{2} < l < (n+1)\ \frac{L_2}{2}$$

5. That the agitation is proportional to the amplitude of the incident wave, even in the case of viscous friction as long as the quadratic friction is considered negligible.

It is relatively easy to obtain approximately the reflection coefficient of a beach in nature, that is, it is easy to find the value of p. It is certainly more difficult to determine the damping of a wave. Damping is particularly important in laboratory scale tests in which the motion in the boundary layer is likely to be laminar. In nature, as a first approximation, this effect could probably be neglected, that is r = 1, except when a wave trap or the presence of lateral beaches could produce important frictional effects.

CONCLUSION

The motion of a wave on a discontinuity has been analyzed previously.[4] In this paper, considering the entrance and the back of a harbor as discontinuities, a theory has been formulated that permits the computation of the value of the agitation in a basin of relatively simple shape. Such an idealized basin is rarely met in practice, but a number of cases may be considered as very similar. This theory is valid to explain not only some phenomena of agitation in harbors but seiche motion and even tidal motion in bays and on the continental shelf.

The main interest of the theory is to show quantitatively in an idealized case and qualitatively in some more complex cases the importance of a number of parameters such as the shape of the entrance, the slope of the terminal beach, and also the importance of multiple reflections and the number of wave lengths contained in a basin. It has been shown that for the case of resonance, the agitation in a basin could become greater inside the harbor than in the open sea.

Only the main topics have been emphasized in this paper, but it is possible by the set of formulas presented herein to study the previously unknown influence of a very great number of other parameters.

The power and relative simplicity of the method of computation by complex numbers in the analysis of a case that, even though idealized, is fairly complicated, should be emphasized.

This theory is limited to the computation of the agitation in a harbor subjected to a wave of constant amplitude and constant period. In the field, waves are irregular. Hence, to be completed this study must be followed by another

in which the probability of obtaining a degree of agitation is to be computed as a function of the incident wave spectrum.

ACKNOWLEDGMENTS

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The author especially thanks R. J. Kennedy for his encouragement. He also thanks J. I. Collins who proof-read this report prior to publication.

APPENDIX.—NOTATION

The following symbols, adopted for use in this paper, are listed here for easy reference and for use by discussers:

$\overline{A}_1$ = complex number of modulus.

A_1 = (semi-amplitude of the incident wave) and argument.

$\hat{A}_1$ = (phase of the incident wave at the entrance to the harbor).

A = shoaling coefficient (see Eq. 28).

A_g = agitation at the back of the basin.

$A_{g_{max}}$ = agitation with resonance.

$A_{g_{min}}$ = minimum agitation, opposite of resonance.

$\overline{B}_1$ = complex number of modulus.

B_1 = (semi-amplitude of the reflected wave seawards) and argument.

$\hat{B}_1$ = (phase of the reflected wave at the entrance to the harbor).

b = length of the entrance.

$\overline{C}_1$ = complex number of modulus.

C_1 = (semi-amplitude of the wave travelling towards the basin at the entrance) and argument-phase.

$\hat{C}_1$ = at the entrance.

$\overline{C}_0$ = complex number of modulus: C_0 and argument: C_0 : half the wave height and phase of the incident wave at the back of the basin, respectively.

$\overline{D}_1$ = complex number of modulus.

D_1 = (semi-amplitude of the wave arriving from the basin to the entrance) and argument-phase.

$\hat{D}_1$ = at the entrance.

$\overline{D}_0$ = complex number of modulus: D_0 and argument $\hat{D}_0$: half the wave height and phase of the reflected wave at the back of the basin, respectively.

h_1 = depth of the open sea.

h_2 = depth of the basin.

$\bar{i}$ = complex number of modulus: $i = 1$, and argument: $\hat{i} = 0$.

K = coefficient of friction.

L_1 = wave length at the open sea and: $m_1 = \frac{2\pi}{L_1}$.

L_2 = wave length in the basin and: $m_2 = \frac{2\pi}{L_2}$.

l_1 = width of the open sea.

l_2 = width of the basin.

l = length of the basin.

n = a positive integer: n = 0, 1, 2, 3 . . . .

p = coefficient of reflection of the back of the basin.

r = coefficient of decay of the wave-height in the basin $r = e^{-\frac{K l}{L_2}}$.

$\bar{r}$ = complex number of modulus: r and argument: $\hat{r}$.

$\hat{r}$ = difference of phase between the waves at the two extremities of the basin.

Z = change of domain coefficient (see Eq. 26).

$\bar{\alpha}_1$ = complex number of modulus: α_1 and argument: $\hat{\alpha}_1$.

α_1 = coefficient of transmission of the wave $\bar{A}_1$: ratio of the transmitted wave height to the incident wave height: A_1.

$\hat{\alpha}_1$ = change of phase of the transmitted wave at the entrance towards the basin.

$\bar{\alpha}_2$ = complex number of modulus: α_2 and argument: $\hat{\alpha}_2$.

α_2 = coefficient of transmission of the wave $\bar{D}_1$: ratio of the transmitted wave height to the incident wave-height D_1.

$\hat{\alpha}_2$ = change of phase of the transmitted wave at the entrance seawards.

$\bar{\beta}_1$ = complex number of modulus β_1 and argument $\hat{\beta}_1$.

β_1 = coefficient of reflection of the wave: $\bar{A}_1$: ratio of the reflected wave height seawards to the incident wave height: A_1.

$\hat{\beta}_1$ = change of phase of the reflected wave height at the entrance seawards.

$\bar{\beta}_2$ = complex number of modulus: β_2 and argument: $\hat{\beta}_2$.

β_2 = coefficient of reflection of the wave: $\bar{D}_1$: ratio of the reflected wave height towards the basin to the incident wave height: D_1.

$\hat{\beta}_2$ = change of phase of the reflected wave towards the basin at the entrance.

λ = length of the beach at the back of the basin.

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ESTUARIAL SEDIMENT TRANSPORT PATTERNS

By H. A. Einstein,[1] F. ASCE, and R. B. Krone[2]

SYNOPSIS

The sediment deposits most characteristic of estuaries are muds which contain a large percentage of clays. These clays give the mud its particular qualities. The transport of these muds is largely determined by flocculation characteristic of suspended clays. The flocculation process is described and analyzed. The process of deposition and scour of the bed of these sediments is also described. All of these processes are finally combined in a description of the process of shoaling in Mare Island Strait, an artifically deepened side arm of San Francisco Bay.

INTRODUCTION

Sediment movements in a large estuary or bay system appear to an interested observer to have several curious aspects. Sediment materials, transported by land drainage, enter an estuary in varying amounts with the inflowing waters. Except during exceptionally high river inflows, little sediment may leave the estuary to the ocean. It is apparent that river-borne sediment is retained by the estuary. During periods of low sediment inflow to an estuary, however, shoaling in channels and harbor areas may occur at much larger rates

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

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than are observed at periods of high inflow rate. The amount of sediment removed for maintenance of navigable channels often exceeds the total sediment inflow to the estuary. That the estuarial sediment movement consists of more than the deposition of the currently river-borne sediments is shown also by varying degrees and locations of turbidity, often visible at the water surface, that occur at times unrelated to the varying river inflow.

The estuarial environment obscures even the simplest information on sediment movement. Tidal fluctuations at the mouth of an estuary or bay are propagated throughout the estuary with velocities and amplitudes that depend on local bottom and shoreline configurations. These tidal changes, often modified by resonance together with river inflow, result in roughly periodic, continuously changing circulation patterns within the estuary. Water current velocities range from zero at slack water to high velocities, particularly at narrows, during the intermediate periods.

Salinity changes are also characteristic of an estuarial environment. During high runnoff periods, fresh river waters may extend to the ocean. During normal or low flows, however, estuarial waters are commonly saline, with salt contents near that of sea water in the estuary and for some distance up inflowing rivers. The salinity often varies with tidal currents, particularly near the river mouth. Regular wind patterns are often characteristic of the coastal areas of which estuaries are a part. In addition to modifications of water depths and currents by wind, even moderate winds generate wave disturbances that seriously alter the character of the sediment transporting medium in shallow water. Extensive shallows formed by accumulated sediment and cut by sparse dendritic drainage channels are characteristic of bays that might by their nature be expected to have an important role in the estuarial sediment regime.

The San Francisco Bay system in California, for example, has all of the characteristics mentioned. Drainage from the entire Central Valley area is discharged at the northeastern end of the bay, and numerous small streams draining the neighboring area enter the bay at widely distributed points. The bay system covers an area of 485 sq miles, and 70% of this area is less than 15 ft deep at mean lower low water. Large areas less than 6fft deep occur. The bay is about 55 miles long, extending both north and south of a narrow mouth. Resonant tidal variations, excited predominantly by diurnal tide, contribute to water interchange between the north and south bays, and river discharges modify the northern current patterns.

Many similar bays exist, and qualitative aspects of the sediment regime in this bay should also be applicable to them. Recent studies of the sediment behavior in the portions of the bay near the entrance of the predominant land drainage have revealed the sediment transport processes that comprise the sediment regime. This paper presents a brief description of the factors that largely determine this regime and the participation of these factors in over-all sediment movement patterns.

THE SEDIMENTS

Samples of sediment from the bottom of points distributed around the bay system indicate that, except at the mouth of the predominant inflow channels, the sediments are composed 50% or more of clay minerals with the largest particle size ranging up to about 100 mv. The mineral composition of the fines supplied by the watershed includes many clay minerals of which most are clas-

sified as illite and mortmorillonite, with smaller amounts of the kaolinite group added. The bay sediment shows the same composition.

These minerals respond rapidly to changes in their hydraulic and chemical environment because of their small size, their large specific surface areas, and their crystal structures. The minerals are commonly thin plates, flexible sheets, or tubes, and range in effective diameters from a few microns to small fractions of a micron. The arrangements of atoms in these minerals are such that the large surfaces of the minerals have a negative charge, and when surrounded by water containing salts, cations accumulate near the surfaces in a cloud. In waters containing small amounts of salt, these clouds are large and interact electrically, preventing the particles from coming close together, Such suspensions are called "dispersed" and are typical of suspensions in many streams. Since small particles settle only slowly, dispersed particles tend to remain permanently in suspension in flowing water, and represent there part of the "wash load."

When these particles are suspended in water containing a gram of sea salt per liter, or more (sea water contains approximately 34 g per l), the effect of the cation cloud is repressed by the abundance of anions in solution, and particles do not repel one another at the larger distances. They can approach one another freely, and an attractive force, that operates only at short range, will predominate and mutually attach the particles. They then stick together, and if successive particle collisions occur, a multi-particle aggregation, or floc, will form. The shapes of clay particles cause the structure of a floc to be open and largely to contain water as does a sponge. The reduced density and the increased size change the hydraulic character of suspended sediment.

FLOCCULATION

The two requirements for the formation of flocs from dispersed suspensions are the reduction of the repulsive forces and the movement of one particle to another. The first requirement, the salinity of at least 1 g per l, occurs in most of an estuarial system to the extent that nearly every collision results in adherence. Collision of particles can occur in several ways: (1) by Brownian motion, or random thermal movements of small suspended particles; (2) by internal shear motion of the water; and (3) by differential particle settling velocities. Since flocculation determines the hydraulic transport character of suspended clayey sediment in an estuarial environment, it is useful to consider the relative importance of these different ways of bringing particles together.

A theory of rapid flocculation was first given by Smoluckowski in 1917-18. This theory, and extensions by later workers, is given by J. Th. G. Overbeek.[3] Only simplified formulas are used here to illustrate the relative importance of the flocculation mechanisms under conditions which cause every interparticle collision to result in a stable multiplet.

The probability of collision from Brownian motion, I, can be approximated by

$$I = \frac{8\,k\,T\,v_o}{3\,\eta} \qquad (1)$$

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[3] "Kinetics of Flocculation," by J. Th. G. Overbeek, Colloid Science I, Elsevier Publishing Co., Amsterdam, 1952, pp. 278-3-1.

in which k represents Biltzman's constant, 1.38×10^{-16} ergs; T denotes absolute temperature, in K; v_o is the number of primary particles per cu cm; and η represents the viscosity, in poises. Suspended sediment concentrations in channels of San Francisco Bay commonly range between a few and 100 parts per million. Taking 0.3 mv as a representative spherical diameter and 15 C as a typical temperature, I = 0.038 per sec. As primary particles join other primary particles and flocs, the total number of multiplets is reduced, increasing the distance between them and reducing the frequency of collisions.

Each time a collision occurs to all particles, the number of particles is reduced to one half of the previous number, but the particle weight is doubled. The rate per unit time at which the particles grow in mass during any part of the growth period is proportional to their size, but inversely proportional to the time between collisions, which is itself inversely proportional to the number of particles or proportional to the weight of the particles. This means that the rate of particle growth is constant, or for the prior given 0.3-mv, original size particles, the growth for any size floc in 26 sec is equal to the mass of an original floc.

The time to build particles of 100-mv average size and of a specific gravity of 1.10 from particles of 0.3-mv diameter and 2.65 specific weight is

$$26\left(\frac{100}{0.3}\right)^3 \frac{(1.10-1)}{(2.65-1)} = 58 \times 10^6 \text{ sec} = 670 \text{ days}$$

while the growth from an average size of 0.3-mv to a floc size of 10 mv takes only 16 hr. This process, which does not require any outside driving force, must be expected to create the smaller floc in a short period, wherein the large flocs will probably appear only if one of the other effects enters the picture. Only a much faster build-up of flocs could actually maintain an average size much larger than 10 mv if the natural breakage of these rather fragile structures is taken into consideration.

Collisions of suspended particles or flocs resulting from internal shear of flowing water can be calculated by finding the probability of one particle passing through a sphere of radius R within which a lasting bond between particles is formed. The simplest spherical radius would be the sum of the radii of the colliding particles. For a water shearing rate, du/dz, and a concentration of particles V, the probability of collision J is

$$J = \frac{4}{3} V R^3 \frac{du}{dz} \qquad (2)$$

A comparison with the probability of collision from Brownian motion yields the ratio

$$\frac{J}{I} = \frac{\eta R^3 \frac{du}{dz}}{2 K T} \qquad (3)$$

The relative importance of internal shear strongly grows with the floc or particle size. For a shearing rate of 1 per sec and a particle or floc diameter of 2 mv, the ratio would equal one. A 10-mv floc would already give J/I = 120. It can be concluded that after flocculation by Brownian motion has begun, internal shear rapidly accelerates flocculation.

An internal shearing rate of 1 per sec probably does not occur in open flow except near the boundaries and in strong turbulence. The bulk of sediment suspended in the flow must form flocs of moderate size, therefore, without significant increase in flocculation rate by shear. Under wave conditions, unstable flow, flow around pilings or other objects, the flocculation rate, enhanced by local turbulence, can "snowball" and produce flocs of sizes that rapidly settle.

When particles having a wide range of sizes are suspended together, larger ones settle faster than smaller ones, and increased frequency of collisions occur. H. Müller[4] studied this mechanism and showed that a significant effect is found when the radius, r, of the settling particle is

$$r \geqq \left[\frac{40\ k\ T}{\eta\ g\ \rho}\right]^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

in which g is acceleration of gravity and ρ is the density of the particles. The term in brackets corresponds to a 5.0-mv diameter solid particle or a 6.3-mv floc. The particles caught must be larger than

$$r \geqq \left[\frac{1.2\ k\ T}{\eta\ g\ \rho}\right]^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

which corresponds to a 2.1-mv solid particle, or to a 2.6-mv floc. It can be seen that this mechanism is most important for collection of small flocs by larger ones. This mechanism intensifies the formation of large flocs after accelerated flocculation by shear, and further enhances deposition near pilings or flow-disturbing works.

DEPOSITION

While the individual clay particles are so small that Brownian motion alone is almost able to keep them in permanent suspension, the flocs of such particles are sufficiently great and heavy to make them settle out in still water. Large flocs have a settling velocity measured in feet per hour. Even the slack water periods in tidal areas are sufficiently long to permit considerable amounts of flocculated clay to accumulate on the bed.

Laboratory experiments show that at a density of about 10 g per l, the clay flocs form a continuous structure. This density is obtained by settling of individual flocs. Settling near this density occurs at a much reduced velocity compared to that at low concentrations, because of the increased resistance of the waters escaping from between the many flocs. One speaks here about "hindered settling." At a density of 10 g per l, the flocs make bodily contact and form a continuous cohesive mass of the same density and strength as the individual large flocs. This mass can support a measurable shear stress without permanent deformation, at least for restricted duration. Upon prolonged stress, it will very slowly creep. Such creep may be either a shear motion or a compaction, or both, depending on the type of stress applied. Compaction is particularly important because it causes the strength of the deposit to increase as more and more bonds are made.

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[4] H. Muller, Kolloidchem, Beihefte, Vol. 27, No. 223, 1928.

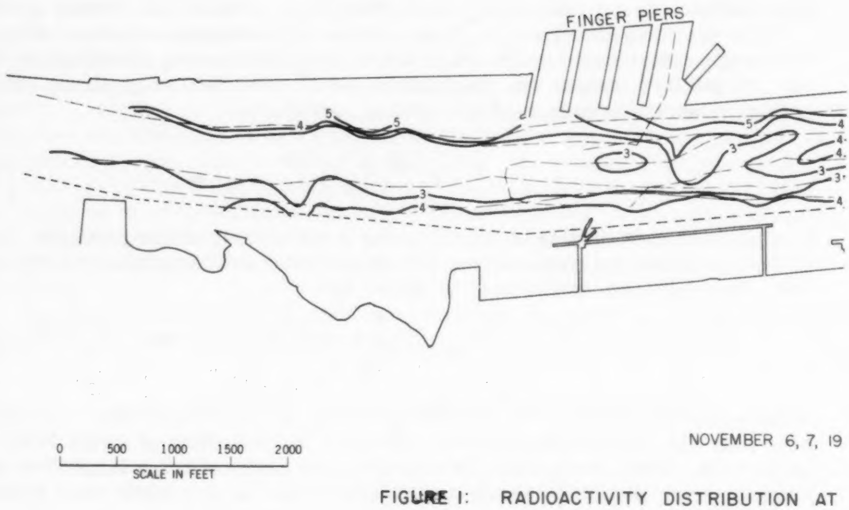
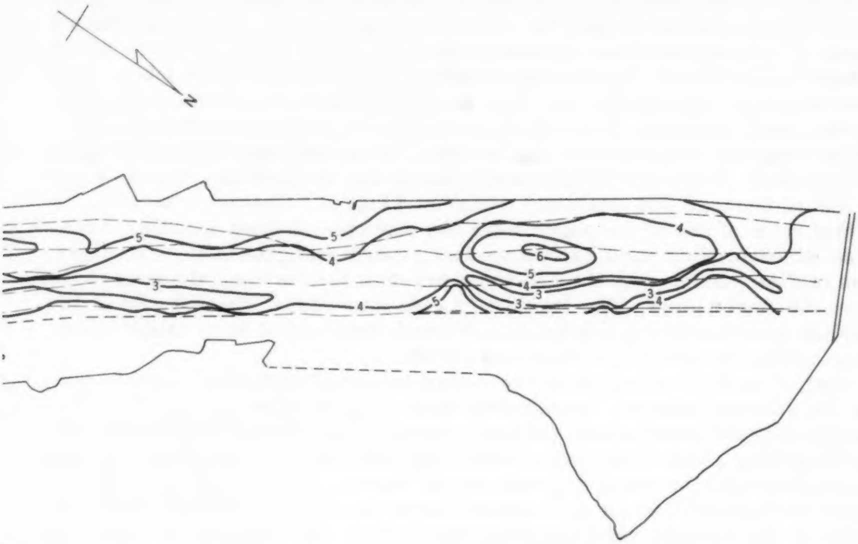


FIG. 1.—RADIOACTIVITY DISTRIBUTION

Deposition occurs not only in still water, but also in flowing water. Up to a flow velocity of 0.5 fps, deposition is the same as in still water, that is, the entire flocs are deposited at and bonded to the bed. As the velocity increases, some of the loosely attached particles are occasionally torn off the bottom while smaller flocs or parts of larger flocs can still deposit on the bed. A denser and tougher layer of clay develops at the bed surface, but at a much slower rate. These stronger bonds between clay minerals can be visualized to develop where they attach their flat sides to one another, in contrast to edgewide contact in the loose flocs. The more flat contacts are made, the stronger becomes the structure. A wide range of such structures occurs in nature, each being characteristic for a particular velocity of flow during deposition. This picture is further complicated by secondary compaction of the deposits due to gravity.

SCOUR

The scour of these clay deposits has not been studied with as much detail as has their deposition. It is unknown whether scour is caused predominantly by the shear stress at the bed or by pressure fluctuations. It has been observed, however, that scour does not occur by chipping large complexes from the bed, but rather by the continuous wearing off of small particles. The speed of scour is reduced with increasing concentration of the flowing water, indicating a type of equilibrium condition to be approached between scour and dep-



9

BOTTOM OF MARE ISLAND STRAIT

AT BOTTOM OF MARE ISLAND STRAIT

osition. No attempt has been made, so far, to define the conditions of such an equilibrium.

FIELD OBSERVATIONS

In returning to the problems of the previosuly quoted San Francisco Bay, some particular observation may be described that resulted from a prototype study which had the purpose of providing a basis for the verification of the Corps of Engineers' hydraulic model of the Bay at Sausalito, Calif. One of the most important problems is the very high rate of mud deposit in Mare Island Strait, the lower end of the Napa River channel, just above its junction with the main Bay system. This junction is at Carquinez Straits, a narrows of the Bay in which the fast tidal currents prevent deposition of mud. The deposited sediment was identified as identical with that of the bay proper, not with that of the Napa River.

The most perplexing observation was made in connection with the time at which the sediment was deposited. It was found that deposition occurred predominantly during the summer months when the bay water contains the lowest sediment concentration and when the rivers carry practically clear water. It was, thus, necessary to find the immediate source of the material somewhere in the bay, even if the material originated from the major river system.

In order to establish the exact process by which the sediment enters Mare Island Strait, a number of specific observations were made. First, a suspended charge of bay sediment was released at the beginning of flood tide at the mouth of Mare Island Strait, after it was identified by radioactive gold label. This trace sediment was found to have been almost fully deposited in the straits during the first tidal cycle. It was found, furthermore, not to have moved significantly during the next few days during which the activity was still sufficient to be identified. Within the Strait, distribution of the radioactivity was the same as that of the normally deposited sediment (Fig. 1). Characteristically, the highest rates of deposition occur on the two sides behind piles and other structures which provide local turbulence that causes fast growth of large flocs. This is shown in Fig. 1 by the 500-count-per-min contour along the west shore above the finger piers. The experiment also showed that the majority of the shoaling material enters in suspension through the mouth of Mare Island Strait, which enters the area from Carquinez Strait.

Next, it must be asked, from where may suspended sediment originate during the summer months. Immediately west (this is, downstream) from the junction of Mare Island Strait, the main channel of Carquinez Strait widens into San Pablo Bay. This large body of water has wide, shallow, tidal flats in which the regular summer winds generate strong waves. These break near the entrance to Carquinez Strait and suspend large amounts of sediment. The turbidity at the surface, resulting from this action, can regularly be observed from the air.

Finally, another test with radioactive tracer was made to study the removal of sediment from the tidal flats during periods of wave action. Bay sediment, labeled with radioactive gold, was placed in suspension above the bottom during a quiet period. The tracer was placed over two shallow areas of San Pablo Bay, and part of the tracer was deposited with the natural sediment. These deposits were established at different depths. Later, a moderate wave condition occurred and it was found to have disturbed the shallow deposit somewhat, but to have left the deeper deposit undisturbed. The disturbance appeared in the form of elongations of the radioactivity pattern not in the direction of the wave action, but in the direction of slow local currents. The same effect had been observed previously in the laboratory. Wave action will suspend the sediment in the water, but does not move it a significant distance. Transport of sediment from the mud flats is predominantly by suspension in moving water.

CONCLUSIONS

During studies of the motion of flocculated sediment in estuaries, both in the laboratory and in the San Francisco Bay system, it was found that the clay sediments flocculate slowly as they advance from the rivers into water with salinity of 1 g per l and more. A faster development of the largest flocs which settle rapidly is caused by a general turbulence which exists in the extended tidal flats. This sediment supply occurs during winter floods.

During almost daily summer winds, some of these winter deposits are stirred up by wave action, and high concentrations of flocculated particles occur. These are moved by local currents into the quiet areas of the artifically deepened channels, particularly where pilings and other harbor structures cause rapid formation of settleable flocs. Here the sediment deposits and necessitates ex-

pensive maintenance dredging programs for the preservation of navigable shipping lanes.

ACKNOWLEDGMENTS

Both the laboratory and the field work described herein are parts of investigations of San Francisco Bay sediment transport processes sponsored by the San Francisco District, U. S. Army Crops of Engineers.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

VIBRATION PROBLEMS IN HYDRAULIC STRUCTURES

By Frank B. Campbell,[1] F. ASCE

SYNOPSIS

The paper is principally concerned with the vibration of control gates and regulating valves commonly used in hydraulic structures, such as dams. The principal findings of 12 yr of vibration investigations by the Army Engineers civil works activities are reported.

The classical theory of vibration is reviewed to select those portions applicable to the subject. Evidence of Coulomb damping is demonstrated and the results of field tests are analyzed to determine the constant friction force. Both laboratory and field tests are reported.

Emphasis is placed on exicting forces expected to be found in hydraulic structures. The Von Karman vortex trail, self-excitation involving reflecting pressure waves and other hydraulic pulsating phenomena are examined. Certain practical conclusions are drawn.

INTRODUCTION

The study of vibration theory has had an interesting history. The physical phenomena involved with musical instruments was of interest to the scientist of the eighteenth century. J. L. Lagrange[2] solved the physical problem.

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

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[2] "Recherches sur la Nature et la Propagation du Son," by J. L. Lagrange, Couvres de Lagrange, Vol. 1, Paris, 1867, p. 39 (Miscellanea Taurinesia, t. 1, 1759).

of the vibrating string in 1759. A great advance to the physics of vibration was contributed in the nineteenth century by Lord Rayleigh.[3] Many refinements to Rayleigh's work have since been accomplished.

It is the purpose of this paper to emphasize to civil engineers, the importance of certain types of mechanical vibrations in hydraulic structures. These particular vibration problems are not adequately covered, even in mechanical engineering literature.

Scope.—Special emphasis is placed on the problems of exciting forces which arise from certain hydraulic phenomena.

The vibration problems arising in connection with the operation of gates and valves commonly employed to control and regulate the flow of water through or over dams will be stressed. The word operation is used because much research needs to be done before a hydraulic structure can actually be designed with confidence in the knowledge that the mechanical elements involved will not vibrate under certain conditions of operation.

The writer is associated with prototype investigations to determine causes of vibrating gates or gate elements. Therefore, this paper will constitute, in a sense, a summary report of the U. S. Army Engineers experiences of undesirable vibration, which have been brought to the attention of the Waterways Experiment Station. The problems of vibration in hydraulic machinery will not be treated, nor will the problems of water hammer and surge tank design.

APPLICABILITY OF CLASSICAL THEORY

It is important to examine the classical theory of vibration to determine which portions of the science are useful to the solution of the specific problems which might be encountered in hydraulic structures. The theory may be found in various publications such as those of C. R. Freberg, E. N. Kemler,[4] J. P. Den Hartog,[5] S. Timoshenko, D. H. Young,[6] M. ASCE, L. S. Jacobsen, and R. S. Ayre,[7] M. ASCE.

A definition of symbols is shown in Fig. 1. The case of undamped free vibrations is used for the purpose of defining basic physical quantities. The device shown is a mass, defined as W/g, with an elastic suspension having a spring constant K, expressed in 16 per in. In this case, assume a cube of steel supported by a tension spring. There will be an initial static elongation δ_s.

If the cube is forceably pulled downward and released, a vertical vibration will ensue. The basic differential equation is

$$\frac{W}{g}\ddot{X} + K X = 0 \qquad (1)$$

The symbol $\ddot{X}$ signifies the second derivative of displacement with respect to time. This derivative is recognized as acceleration. The first term is,

3 "The Theory of Sound," by J. W. S. Rayleigh, First Amer. Edition, New York, Dover, 1945.

4 "Elements of Mechanical Vibrations," by C. R. Freberg and E. N. Kemler, Second Edition, John Wiley and Sons, Inc., New York, 1949.

5 "Mechanical Vibrations," by J. P. Den Hartog, Fourth Edition, McGraw-Hill Book Co., Inc., New York, 1956.

6 "Vibrations Problem in Engineering," by S. Timoshenko and D. H. Young, Third Edition, Van Nostrand, New York, 1955.

7 "Engineering Vibrations," by L. S. Jacobsen and R. S. Ayre, First Edition, McGraw-Hill Book Co., Inc., New York, 1958.

therefore, the product of mass and acceleration, which represents the inertial force of the vibrating body. The second term has the product of the spring constant and the displacement, which is referred to as the restoring force.

The useful solution to the basic equation is

$$X = X_o \cos \sqrt{\frac{K g}{W}}\ t \quad \text{.............. (2)}$$

One period of the cosine curve is 2π or $\omega_n t$. The subscript n is used to denote the natural frequency of the elastic system. The symbol ω is commonly called the angular frequency although various writers use different symbols.

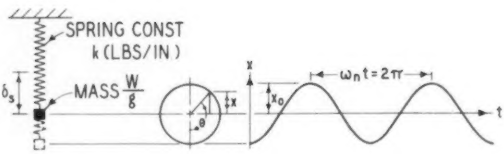

FIG. 1.—DEFINITIONS-UNDAMPED FREE VIBRATIONS

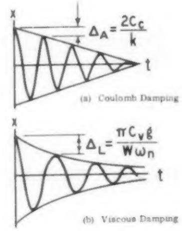

FIG. 2.—DAMPED FREE VIBRATIONS

The natural period is

$$T = \frac{2\pi}{\omega_n} \quad \text{.................... (3)}$$

Since the frequency in cycles per second is the reciprocal of the period,

$$f_n = \frac{\omega_n}{2\pi} \quad \text{..................... (4)}$$

Thus

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_s}} \quad \text{.............. (5)}$$

It may be readily seen that the natural frequency is inversely proportional to the square root of the initial static elongation, δ_s. For the elastic system described, this value is expressed as

$$\delta_s = \frac{W}{K} \quad \text{................ (6)}$$

and

$$T = 2\pi\sqrt{\frac{\delta_s}{g}} \quad \text{............... (7)}$$

The spring constant is thus seen to be an important characteristic of the vibratory system. Some sources give tabulations of the spring constant for various types of elastic systems.

The phase angle circle is included to demonstrate the initial displacement position. Certain writers, notably Jacobsen and Ayre,[7] have developed graphical phase plane solutions which appear to be useful in certain types of damped vibrations.

Damped Free Vibrations.—When a restraining force is imposed on a vibrating system, the type of damping must be known before analysis. The basic differential equations for the three common types of damping are:

Constant friction (no single complete solution)

$$\frac{W}{g}\ddot{X} + K\,X + C_c\,(\sin\dot{X}) = 0 \qquad (8)$$

Viscous damping

$$\frac{W}{g}\ddot{X} + C_v\,\dot{X} + K\,X = 0 \qquad (9)$$

with the solution:

$$X = e^{-\frac{C_v\,g}{2\,W}\,t}\,(A\cos\omega_n t) \qquad (10)$$

Turbulent damping (no direct simple solution)

$$\frac{W}{g}\ddot{X} + C_T\,(\dot{X})^2 + K\,X = 0 \qquad (11)$$

Their applicability to the problem of vibration in hydraulic structures is important to this paper. It should be noted that each of the differential equations contains inertial force involving mass and acceleration plus restoring force involving the spring constant and displacement. To each of the equations a friction or damping force is added.

The constant friction force has the sign of the velocity. The force always opposes the motion and, thus, changes sign at each half cycle. Therefore, no single complete solution can be written (Eq. 8). Each part of the solution must be considered between the largest positive and negative displacements of each half cycle.

The second term of the viscous damping equation (Eq. 9) contains the damping coefficient which is multiplied by the first power of the velocity. The general equation has a relatively simple and complete solution. The first factor on the right contains an exponential function which involves the damping constant C_v and mass. The second factor contains a cosine function of the natural frequency. A large part of the classical theory of vibration is based on viscous damping. Unfortunately, the viscous damping solution does not seem to be generally applicable to problems encountered in high head hydraulic structures.

Turbulent damping (Eq. 11) is the third case to be considered. The turbulent damping force is a function of the square of the velocity. This is the common type of force with which hydraulic engineers ordinarily deal in their problems of pipe friction loss. Unfortunately, the equation is a second order, second degree differential equation and has no direct simple solution. W. E. Milne[8,9] has obtained an elaborate exact solution. He has also prepared tables for the evaluation of the solution which have been found useful in surge tank problems

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8 "Damped Vibrations," by W. E. Milne, Univ. of Oregon Publication, Vol. 2, No. 2, 1923.

9 "Table of Damped Vibrations," by W. E. Milne, Mathematical Series, Vol. 1, No. 1, Univ. of Oregon, March, 1929.

for hydro-electric power plant design. The technique involved is beyond the scope of this paper.

The problem of attenuation of displacement in damped free vibrations is considered next. Constant friction is characterized by an arithmetic decrement as indicated in Fig. 2(a). It is important to note that only the constant friction force and the spring constant are involved in the determination of the arithmetic decrement. This is an important relationship as will be shown later.

In the case of viscous damping, the attenuation for each cycle is called the logarithmic decrement and can be evaluated when the viscous damping coefficient, the mass, and the frequency are known. The attenuation is affected by an exponential law so that the displacement is damped rather rapidly in the beginning, as demonstrated on Fig. 2(b). After the damping factor becomes negligible, consider that a steady state exists. Reference will be made to steady state vibrations, subsequently.

Forced Vibration.—The aspect of a damping force which decreases the energy of the vibrations has been considered. It is now appropriate to consider

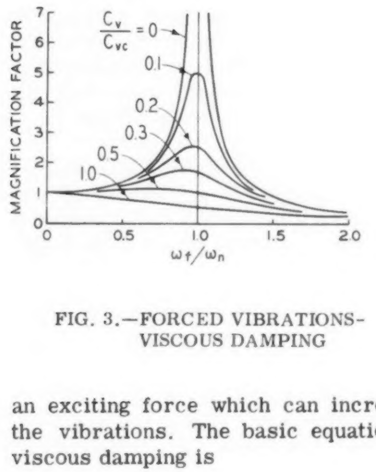

FIG. 3.—FORCED VIBRATIONS-VISCOUS DAMPING

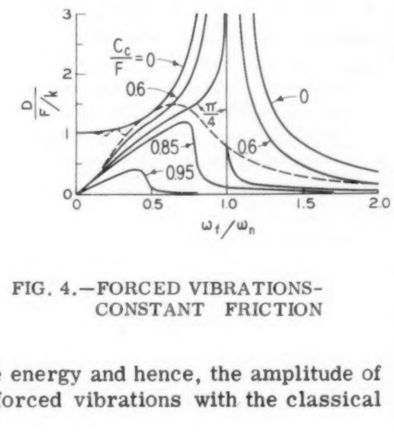

FIG. 4.—FORCED VIBRATIONS-CONSTANT FRICTION

an exciting force which can increase the energy and hence, the amplitude of the vibrations. The basic equation for forced vibrations with the classical viscous damping is

$$\frac{W}{g}\ddot{X} + C_V \dot{X} + K X = F \cos\left(\omega_f t\right) \quad \text{.............. (12)}$$

The solution shown in Fig. 3 is for the case of steady state vibration. In other words, the damping effect of the exponential term has become negligible. The exciting force F is considered to be applied with a cosine variation. The solution for the steady state vibration is

$$X = \frac{F}{K} \frac{\cos\left(\omega_f t - \theta\right)}{\sqrt{\left[1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left(2 \frac{C_v}{C_{vc}} \frac{\omega_f}{\omega_n}\right)^2}} \quad \text{........ (13)}$$

The first factor of the solution is the ratio F/K, which represents the static elongation that would be produced by the maximum exciting force with the existing spring constant. If the cosine function in the numerator is placed equal to 1, the factor one would be divided by the radical, which is ordinarily known as the Magnification Factor. Note that the Magnification Factor involves ratios of forcing frequency to natural frequency for the various damping ratios C_v/C_{vc}. The denominator C_{vc} of this ratio is the critical damping coefficient which can be evaluated according to

$$C_{vc} = 2\,\omega_n \frac{W}{g} \qquad (14)$$

The critical damping coefficient involves only the natural frequency of the vibrating system and its mass.

In considering the subject of damping friction, it would be appropriate to quote from Jacobsen and Ayre[10] as follows: "Friction is therefore, a priori, a more complicated property to deal with than either inertia or restoration."

A graph can be constructed of the Magnification Factor as a function of the frequency ratios. The various damping ratios are represented by a family of curves. For a damping ratio of zero, the curve represents an undamped system. Near the condition of resonance where the ratio of frequencies is equal to one, the Magnification Factor and the amplitude approaches infinity. The system is safe against wildly fluctuating vibrations in the resonant range if the damping ratios are substantially greater than 0.3.

Constant Friction.—As previously mentioned, the differential equation for the case of constant friction has no simple continuous solution. Nevertheless, Den Hartog[11] developed a remarkable solution for maximum displacement in the case of steady state vibration with constant friction. His equation for maximum displacement is

$$D = \frac{F}{K}\sqrt{A^2 - \frac{C_c^2}{F^2}\,B^2} \qquad (15)$$

in which

$$A = \frac{1}{1 - \dfrac{\omega_f^2}{\omega_n^2}} \qquad (16)$$

and

$$B = \frac{\omega_n}{\omega_f}\tan\frac{\pi\,\omega_n}{2\,\omega_f} \qquad (17)$$

The solution again contains a ratio of exciting force to spring constant. The variables A and B under the radical are two different functions of the ratio of forcing frequency to natural frequency of the system. The second term under the radical contains a square of the ratio of the constant friction force to the

10 "Engineering Vibrations," by L. S. Jacobsen and R. S. Ayre, First Edition, McGraw-Hill Book Co., Inc., New York, 1958, p. 196.

exciting force. The magnification can be expressed as the ratio of the maximum displacement to the static elongation attributable to the exciting force.

$$\text{Magnification} = \frac{D}{\frac{F}{K}} \qquad (18)$$

When the second term under the radical (Eq. 15) becomes larger than the first, the evaluation of maximum displacement involves the square root of a negative number. Eq. 15 is only of value in the determination of magnification above the dashed line shown in Fig. 4. The family of curves represent various ratios of the constant friction force to the maximum exciting force. For a ratio of forces greater than $\pi/4$, the magnification approaches infinity.

By a complex procedure, Den Hartog[11] was also successful in evaluating single points underneath the dashed line and he constructed curves representing high force ratios. The friction is sufficient in the region of high force ratios to cause a stoppage in the vibratory motion. It is interesting to note that the two lines representing force ratios of 0.85 and 0.95 from the Den Hartog investigation have maxima at frequency ratios substantially smaller than the resonant frequency. It might be expected that, in the case of constant friction, if the force ratios are considerably greater than 0.8, there would be little likelihood of developing large vibratory amplitudes. In the case of viscous damping, we surmised that the situation would be fairly safe if the damping ratios exceeded 0.3.

The engineer is then faced with the problem of determining the constant friction force and the exciting force. Some test data are available on friction factors, which should be applicable to hydraulic gates. On the other hand, experimental data on the magnitude of exciting forces in the various hydraulic phenomena are practically nonexistant. The problem of excitation in hydraulic structures will be presented later.

Hydraulic Gate Friction.—A substantial amount of work has been conducted on sliding friction and rolling friction research. However, much of the recent experimentation has been concerned with the effect of various types of lubrication on journal friction. It is believed that ordinary lubricants are not effective over a long period of time when used on submerged high head hydraulic gates.

The U. S. Bureau of Reclamation[12,13] has conducted tests on both sliding and rolling friction, in connection with their studies of gates for hydraulic structures. These laboratory tests were conducted in the dry rather than with water surrounding the element as would be expected in an outlet works, for example. It appears that the coefficient for sliding friction is in the neighborhood of 0.1, whereas the coefficient of rolling friction may be approximately 0.001 depending on the type of metals used. In the field of hoist design for high head gates, provision must be made for adequate capacity to break the seizure of metals after the gate has been closed under a high head for a substantial period of time. Therefore, with application of a judiciously chosen factor of safety, the coefficients mentioned would probably be entirely ade-

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11 "Forced Vibrations with Combined Coulomb and Viscous Friction," by J. P. Dem Hartog, Transactions, ASME, Paper APM 53-9, presented at the June 1931 National Convention of Applied Mechanics at Purdue Univ. in Lafayette, Ind.

12 "Tests on Rollers," by N. G. Noonan and W. H. Strange, U. S. Bur. of Reclam., Tech. Memorandum No. 399, Separate No. 26, September 26, 1936.

13 "Report of Tests on Coefficients of Friction," by N. G. Noonan and W. H. Strange, U. S. Bur. of Reclam., Tech. Memorandum No. 432, January 30, 1935.

quate for purposes of hoist capacity design. On the other hand, it is believed that further investigation should be made of friction forces in the prototype to determine a more accurate value for use in the problems of gate vibration.

GATES ON ELASTIC SUSPENSIONS

Model Studies.—Hydraulic model studies of a mass on an elastic suspension have been found to be practical and useful in a study of exciting forces. The Waterways Experiment Station[14] made studies for the Omaha District beginning in 1947. These studies simulated the early Garrison control gate design as well as the Fort Randall control gates.

The mass of the gate in the prototype was simulated in the model. An extension spring was selected which produced a static elongation similar to that expected in the prototype. Considerable care was taken to reduce friction in the model by the use of minature roller bearings. Both the vibratory motion and the forces on the gates were measured.

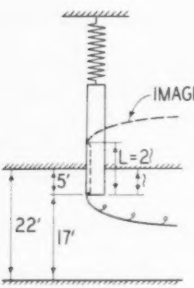

FIG. 5.—FLAT BOTTOM GATE-VORTEX TRAIL RESONANCE

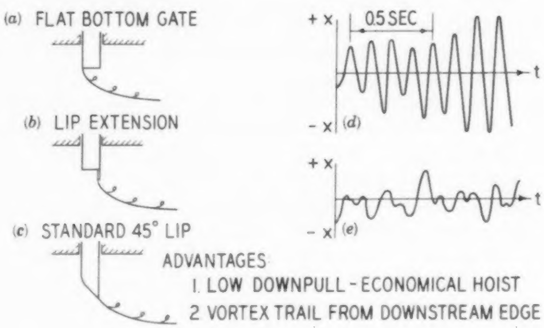

FIG. 6.—DEVELOPMENT OF STANDARD 45° LIP

One of the objectives of the tests was to determine whether a vortex trail from a partly open flat bottom gate could produce vibrations. The basic theory of this investigation is demonstrated in Fig. 5. The initial static elongation of the model gate on its elastic suspension $\delta = 0.0126$ ft. Substituting Eq. 5, the natural frequency of this system $f_n = 8$ cycles per sec.

When a flat plate is placed laterally across a stream, the Von Karman vortex trail is shed from each edge of the plate. The frequency of the shedding of these vortices is defined by the Strouhal number (S_t) which involves the frequency, the length of the plate, and the velocity of the fluid.

$$S_t = \frac{f_f \quad L_m}{V_m} \qquad (19)$$

14 "Spillway and Outlet Works, Ft. Randall Dam, Missouri River," Tech. Report No. 2-528, U. S. Army Engrs., Vicksburg, Miss., October, 1959.

Fig. 5 shows the flat plate to have half of its length protruding into a gate. chamber. This represents the partial opening of a control gate. The image of the flat plate above the roof of the conduit completes the normal concept of the vortex trail. During the early phases of the tests it was not known whether, the vortex trail can be shed from half of a plate as shown in this situation. However, it was soon learned that this phenomena can exist for the protrusion of a gate leaf into a conduit.

Various experimenters[15] have determined the Strouhal number for a flat plate normal to the flow to be approximately 1/7. If the vortex trail can produce an exciting force on the bottom of the gate which would come into resonance with the natural frequency of the elastic system, it would be expected that the gate would produce vibrations of large amplitudes. For a 17-ft prototype gate opening, L_m = 0.333 ft, and a model velocity V_m = 18.5 fps, the theory indicated that there would be essential resonance. For a normal plate $S_t = 1/7 = 0.143$ and $f_f = 7.92$, thus the ratio of forcing frequency to natural frequency is

$$\frac{f_f}{f_n} = \frac{7.92}{8.0} = 0.99$$

It was actually determined that when the model gate was set to simulate a 17-ft prototype gate opening, violent vertical vibrations were produced. Therefore, it was obvious that resonance between the frequency of the vortex trails from a gate projection acting as an exciting force on an elastic system could actually be produced in the laboratory.

Split Leaf Gates.—In connection with the subject of the vortex trail as an exciting force, mention should be made of split leaf spillway gates. With this system, one leaf rests on top of the other. In times of flood-flow both leaves are removed.

After the top leaf has been removed and the bottom leaf is in the process of raising, water can flow over and under the partially raised leaf. The full vortex trail is then effective in producing periodic pulses, alternately, on the top and bottom of the gate leaf.

The Waterways Experiment Station studied this problem with a model of the Old River Control Structure gates.[16]

Improvement of Gate Lips.—It was learned from the studies of Ft. Randall gate vibration that a flat bottom gate had high downpull. The tests at the Waterways Experiment Station further showed that a flat bottom gate also was more susceptible to vibtations. Various lip extensions from the downstream edge were tested. The flat bottom gate and the lip extension schemes are shown in Fig. 6(a) and (b). Typical oscillograms translated in terms of vertical displacement from the accelerometer records are shown in Fig. 6(d) and (e). A fairly regular motion of 8 cycles per sec may be seen in Fig. 6(d), whereas no such regularity is shown for the case of a lip extension (Fig. 6(e)).

The vortex trail was probably springing essentially from the lip extension rather than from the upstream edge of the flat bottom gate. The engineers of the Waterways Experiment Station tried the next logical step which was a gate with a 45° bottom surface (Fig. 6(c)). It was found that the vortex trail would

15 "Modern Developments in Fluid Dynamics," by S. Goldstein, First Edition, Oxford, Clarendon Press, Vol. 2, 1938, p. 571.

16 "Downpull Forces on Vertical Lift Gates," Tech. Report No. 2-477, Report No. 1, U. S. Waterways Experiment Sta., Vicksburg, Miss., December, 1956.

then spring from the downstream edge and minimize pulses on the bottom of the gate.

No tendency for vibration was found for the 45° gate lip. Furthermore, the downpull or rather the reduction of pressure on the bottom of the gate was much less for the sloping gate bottom than for the flat bottom. Therefore, the 45° gate lip has become standard on Army Engineer gates because reduced downpull is indicated and there is less tendency towards vibration of the gate.

Prototype Tests.—Before proceeding with a report of the prototype tests, it is appropriate to briefly examine the important problem of the spring constant for cable supported control gates. Tests were made by Waterways Experiment Station engineers at Knightville Dam in New England in the fall of 1955. In the analysis of the data, it was desirable to compute the spring constant of the cable supports. If the total metallic cross-sectional area of the cable and the modulus of elasticity are known, the spring constant can be determined. A fairly accurate value of metallic cross-sectional area can be furnished by the manufacturers of wire rope.

The only information on modulus of elasticity of wire rope at that time was a value of 12,000,000 psi for unstressed cables furnished many years ago by one of the wire rope manufacturers. The use of this value did not produce a theoretical frequency which agreed with the measured frequency at Knightville Dam on tests at three different gate openings ranging from 10 1/2 ft to 11 ft.

The natural frequency measured at Knightville was about 6 1/2 cycles per sec. Knowing the other physical data pertaining to the vibrating system, the modulus of elasticity was estimated to be approximately 21,000,000 psi. Soon thereafter, information became available from tests on prestressed wire rope. These results indicated that the modulus of elasticity should be in the neighborhood of 20,000,000 psi. This value is now considered to be a fairly reliable modulus of elasticity for wire rope which has been stressed repeatedly in operation.

It was clearly evident that the mass of the water contained within the gate must be added to the mass of the gate metal for a proper vibration analysis.

Fort Randall Tests.—The Waterways Experiment Station[17] conducted a series of three tests, for the Omaha District, on the control gates for Ft. Randall Dam in 1954 and 1955, while the reservoir was filling. Fig. 7 pertains to the test made with a head of 110 ft. The spring constant for the cable suspension was 1.7×10^5 psi and the gate weighed approximately 47 tons. Gate roller tracks and side guides are shown schematically. The side guides restrain any large motion of the gate in the upstream direction. Half of the guides were spring mounted and the other half were not.

The oscillograph records show that for large gate openings, there were occasional vertical vibrations of approximately 4 cycles per sec. These vibrations continued for several cycles and were then damped out. After an examination of the record, it was apparent that the frictional force involved in the rollers was rather high, but that occasionally a surge against the downstream face of the gate moved the gate upstream so that momentarily the rollers were not in contact with the track. During these intervals the gate was permitted to vibrate vertically. As the rollers again came in contact, the vibration was damped. Further examination of the damping periods of the record indicated

17 "Vibration and Pressure-Cell Tests, Flood Control Intake Gates, Ft. Randall Dam, Missouri River," Tech. Report No. 2-435, U.S. Waterways Experiment Sta., Vicksburg, Miss., June, 1956.

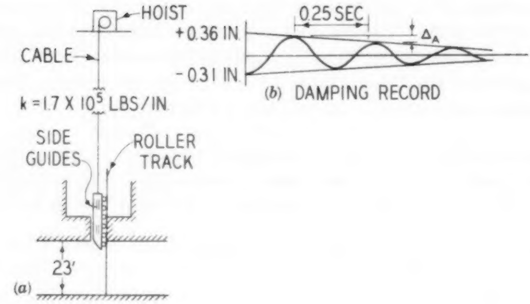

FIG. 7.—ELASTIC SUSPENSION-COULOMB DAMPING

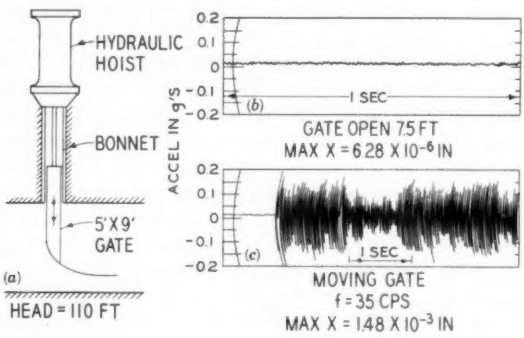

FIG. 8.—HYDRAULICALLY OPERATED SLUICE GATE

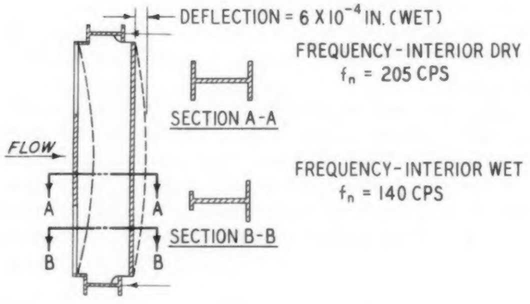

FIG. 9.—CONTROL GATE-LATERAL VIBRATION

that the damping was of a Coulomb character. One of the damping records is shown in Fig. 7(b). It may be noted that straight lines connect the peaks and troughs of the curve.

This was a significant finding, for although experimental evidence was at hand to show that the damping was predominantly caused by a constant friction force, the theoretical problem was known to be complex. It is pertinent to quote again from Jacobsen and Ayre[18] as follows:

> "It is therefore clear that the presence of constant friction greatly complicates the analysis of steady state response to an alternating force, especially if the friction is large enough to enforce stops in the motion."

The arithmetic decrement was then computed for several damping periods. Knowing the spring constant and the arithmetic decrement, the damping force (C_c) can be computed.

$$C_c = \frac{K \Delta_A}{2} \quad \text{. . . . .} \quad (20)$$

The section of the record shown in Fig. 7(b) is for a 20 ft gate opening and the damping force was computed to be 2,740 lb. An average of four observations (gate openings of 19 ft to 21 ft) indicated a damping force of about 2,800 lb. It should be mentioned that in two cases studied for the same gate opening, one damping force appeared to be double the damping force for another period. It is possible that the lower force could have been caused by only one roller track coming in contact with the guide. In the case of the higher force, it appears that both roller tracks came in contact with the guides simultaneously.

It was realized that although the theory of Coulomb damping is complex, the simple determination of the arithmetical decrement affords a means of measuring friction. It is believed that a study of damping records can furnish valuable information on the total damping force for any particular system under observation. There has not been an opportunity to attempt to compare the total damping force measured in the prototype with a computed force based on friction factors as measured in the laboratory. An estimation of the frictional forces of the individual components of a roller train may not be a simple matter.

Sluice Gates.—The hydraulically operated sluice gate is a common device used for controlling the flow of water through concrete dams. Prototype tests were made on the sluice gates of Pine Flat Dam, Calif. for the Sacramento District in 1952. Measurement of the vibrations of the gate leaf was a part of these tests. The gate size was 5 ft by 9 ft. Fig. 8(a) shows the schematic section of the slide gates.

The 6 ton sluice gate was supported by a 6 1/2 in. diameter steel stem. A typical oscillogram is shown in Fig. 8(b). The maximum amplitude of vibration was of the order of 6/1,000,000 in. This emphasizes the fact that we are dealing with a relatively light weight gate on a stiff suspension.

The oscillogram in Fig. 8(c) represents a significant observation on the Pine Flat sluice gate. This record represents the vibration existing when the gate was being moved from a 3 ft to a 4 ft opening. For such a situation, the

18 "Engineering Vibrations," by L. S. Jacobsen and R. S. Ayre, First Edition, McGraw-Hill Book Co., Inc., New York, 1958, p. 226.

amplitude exceeded 1/1000 in. or a thousand times that observed for the stationary position. It seems reasonable to believe that this larger vibration amplitude was caused by alternate seizure and relief of the metals on the slide bearing. This is sometimes called chatter.

Although the hydraulic head for the test just described was 110 ft, this gate was subsequently operated under a head in excess of 300 ft. No detectable damage to the gate was in evidence. A cursory study was made of the fatigue aspect in the gate stem. The stresses in the stem proper were so low as to eliminate any further consideration of fatigue. No study has been made of the possible fatigue aspects of the supporting elements at either end of the stem.

ELASTIC BEAMS

Flexural Vibration.—The horizontal beams of a tractor gate are normally stiff. The Fort Randall gate beam is shown on Fig. 9.

For a concept of the stiffness, it may be noted that the initial static deflection based on the mass of the gate and contained water is only 6×10^{-4} in. A comparison is shown between the theoretical frequency of the beam with an without contained water.

The frequency for the dry condition may be seen to be 205 cycles per sec and 140 cycles per sec for the condition of inclosed water. This emphasizes the importance of accounting for the mass of the contained water for gates of this type.

PLATES AND SHELLS

Howell-Bunger Valves.—Some years ago it was learned that weld seams on 4-vane Howell-Bunger valves at two different projects had failed. Fig. 10 is a schematic representation of the vibration characteristic of both a 4-vane and a 6-vane valve shell. Timoshenko treats the subject of vibration of thin walled elastic cylinders.[6]

Fig. 10(a) shows schematically a 4-vane valve shell vibrating with the fundamental mode and Fig. 10(b) a 6-vane valve shell vibrating with a secondary mode. Eq. 21 gives the natural frequency for the fundamental mode and any of the higher modes.

$$f_i = \frac{1}{2\pi} \sqrt{\frac{E g I}{\gamma A r^4}}\, M_i \qquad (21)$$

It may be seen that the factor involving the radical is concerned with the mass and elasticity of the shell. The factor (M_i) is designated a mode factor.

$$M_i = \sqrt{\frac{i^2 (1 - i^2)^2}{1 + i^2}} \qquad (22)$$

For the fundamental mode (Fig. 10(a)), i = 2, and the resulting mode factor M_2 = 2.68. For the secondary (Fig. 10(b), i = 3, and the value of the mode factor M_3 = 7.59. Since the natural frequency of the shells are proportional to ratio of the fundamental mode is 2.82.

$$f_2 = \frac{2.68}{2\pi}\sqrt{\frac{E\,g\,I}{\gamma A r^4}} \qquad (23)$$

and

$$f_3 = \frac{7.59}{2\pi}\sqrt{\frac{E\,g\,I}{\gamma A r^4}} \qquad (24)$$

Thus

$$\frac{f_3}{f_2} = 2.82$$

This ignores the existence of vanes within the shell. However, it appears that by the use of 6 vanes and producing a vibration of the secondary mode type,

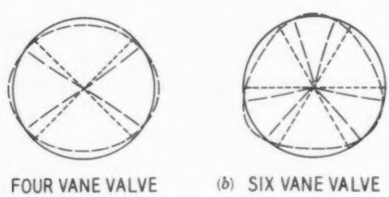

FIG. 10.—HOWELL-BUNGER VALVE VIBRATION

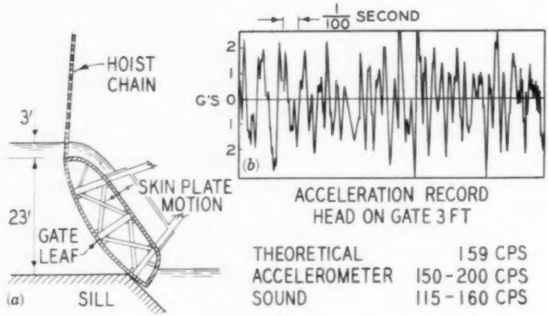

FIG. 11.—SKIN-PLATE VIBRATION—SUBMERGIBLE GATE

the frequency could be expected to be much higher than for the 4 vanes and the fundamental mode. When the frequency is substantially higher, it would normally be expected that the amplitude of vibrations would be lower and that for each cycle the total amount of energy is less for the higher modes. For this reason, it is believed that a 6-vane valve is less liable to know failure of the seams which connect the vanes to the shell.

To the writer's knowledge, the problem of the Howell-Bunger valves with the mass and elastic characteristics of both shell and vanes has not been treated. It would be interesting indeed to see a thorough analysis of the 4-vane valve compared with the analysis of the 6-vane valve. Furthermore, it is conceivable that a 5-vane valve would cause the vibration to go to a still higher mode and hence have less liability for failure of weld seams.

Rayleigh-Ritz Method.—It is appropriate to mention the Rayleigh-Ritz method by which problems of this nature can be solved. Lord Rayleigh[3] treated only the case of the fundamental mode. He devised the method of writing the basic equations for both potential energy and kinetic energy. By equating these two energies and by proper manipulation of the equation, he determined the frequencies of the various types of elastic systems. Some years later Walther Ritz[19] extended Lord Rayleigh's method to treat the problems of the vibration characteristics with modes higher than the fundamental. Since that time, numerous contributions have been made to the theory of vibration of elastic elements using the Rayleigh-Ritz method.

Gate Skin Plates.—The vibration of the skin plate on a submergible gate at Cheatham Dam on the Cumberland River offers an example of this type of vibratory motion. The operators reported a loud noise issuing from the Cheatham gate when the overflow head was about 3 ft. The gate is shown in Fig. 11.

The Waterways Experiment Station investigated the cause of the noise in cooperation with the Nashville District. Accelerometers were placed to measure the principal freedoms of motion of the gate itself as well as those of the upstream and downstream skin plate. The specific source of the noise was not known at that time.

It was found that the frequency of the downstream skin plate approximated the frequency of the sound. The frequencies of other freedoms of motion were substantially different. It was therefore concluded that the downstream skin plate was vibrating and causing the sound.

The resonant air chamber inside the gate was analyzed but found to have a low frequency in comparison. Similarly, the vortex trail from the strut arms possessed a low frequency. As the exciting force of the vibratory motion could not be readily determined, it was recommended that the skin plate be restrained by a support in the middle of the panel. It was reasoned that this would greatly increase the natural frequency of the skin plate and eliminate the vibration which emitted the noise.

EXCITING FORCES

Self Excited Oscillations.—The action of the bow on a violin string is the classic example often cited. A good example of self-excitation from the hydraulic engineering viewpoint is the interaction of a reflecting pressure wave a conduit and the vertical vibration of a gate. This is shown schematically in Fig. 12.

A small vertical displacement of the gate would cause a pressure wave to travel upstream in the tunnel. When the wave reaches the reservoir, it is reflected and changes from a positive pressure wave to a negative pressure wave or vice versa.

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19 "Gesammelte Werke," by Walther Ritz, Paris, 1911.

If the speed of the wave is a and the distance from gate to reservoir is L, the period of the pulsation against the gate bottom would be (4 L/a sec. The pressure pulses on the gate bottom are shown on Fig. 12 (b) as a square wave for the sake of simplicity. When the natural period of vertical vibration of the gate is close to the natural period of the pressure wave pulse, a condition of self-excitation could exist.

It is interesting to note that the Von Karman vortex trail is a self excited oscillation in itself.

Free Water Surface Phenomena.—Two free water surface phenomena are shown in Fig. 13. Either could constitute an exciting force on an elastic structural element.

The fluctuating nappe for a small head on a sloping weir has been noted by various observers. This phenomena was extensively observed and analyzed by Bruno Leo.[20] The phenomena was also observed by the Bureau of Reclamation[21] on the drum gates at Black Canyon Dam. In this case, the vibration was eliminated by aerating the space under the nappe.

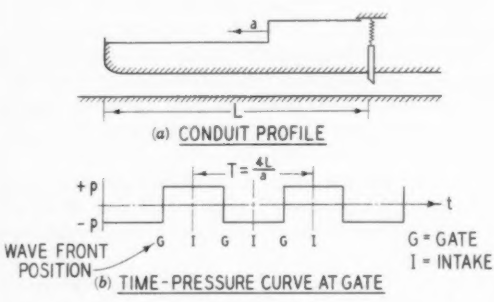

FIG. 12.—SELF-EXCITATION WITH PRESSURE WAVE

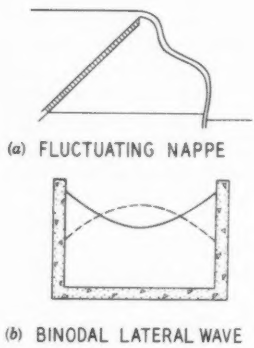

FIG. 13.—EXCITING FORCES—FREE WATER SURFACE

Channel Waves.—The binodal wave which has a clapotis type action from each side wall was observed in the fish ladders at McNary Dam. A trinodal wave was also observed in the Bonneville fish ladders. These waves can constitute an exciting force on adjoining structures.

Other Exciting Forces.—Various other phenomena can possibly constitute exciting forces in vibratory motion. The successive formation and collapse of vapor cavities in the phenomenon of cavitation seem to have a periodicity. Little experimental information is available for the multitude of possible geometrical boundary situations. Some experimenters have suggested a Strouhal type number which defines the frequency of shedding of cavities.

20 "Self Excited Vibrations at Overflow Weirs (Selbstgestenerte Schwengungen an Uber Stromten Wehren)," by Bruno Leo, U. S. Waterways Experiment Translation No. 55-6, Vicksburg, Miss., June, 1955.

21 "Report on Vibration Studies Made at Black Canyon Dam," by R. E. Glover, C. W. Thomas, and T. F. Hammett, U. S. Bur. of Reclam., Hydr. Lab. Report No. 58, Denver, Colo., July 24, 1939.

The fixed cavity which forms on the trailing side of an obstruction in high velocity flow has an intermittent pressure pulse. However, little research has been conducted on this problem.

The toe of the hydraulic jump is known to pulse in an up and downstream direction. The possible effect of this is unknown.

CONCLUSIONS

1. The modulus of elasticity of wire rope suspensions can be determined by measurement of vertical vibrations.
2. The type of friction which affects the damping of hydraulic gates has been shown to be constant friction or Coulomb damping.
3. The magnitude of the constant friction force can be evaluated by an analysis of the oscillograms.
4. The Von Karman vortex trail has been observed to be the exciting force which can cause vibration of gates.
5. A gate leaf with a sloping bottom sheds the vortes trail from the downstream edge and therefore minimizes vibratory motion.
6. Extensive research is needed on the magnitude of the exciting forces before gates can be designed against vibration.
7. Considerable research is needed on the character of vibration of both fixed and traveling cavities and other hydraulic pulsating phenomena.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the supervision of field work by Ellis B. Pickett, A. M. ASCE, Chief of the Prototype Section, and by his predecessor, Benson Guyton, M. ASCE. Most of the electronic instrumentation was conducted by Leiland M. Duke and George C. Downing of the Instrumentation Branch at the Waterways Experiment Station. It was only with the cooperation of many engineers in the Division and Districts of the Army Engineers, that the field tests were possible. Engineers of the Waterways Experiment Station, too numerous to mention, also participated in this work. E. P. Fortson, Jr., F. ASCE, is Chief, Hydraulics Dibision and J. B. Tiffany, Jr., F. ASCE, is Technical Director, Waterways Experiment Station. The field work was performed over a period when C. H. Dunn, F. ASCE, A. P. Rollins, Jr., F. ASCE, and E. H. Lang, F. ASCE, were successively directors at the Station.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

STREAM-GAGING NETWORK IN THE UNITED STATES[a,b]

By John E. McCall,[1] F. ASCE

SYNOPSIS

The national stream-gaging program is analyzed with regard to size, cost, comparison with other countries, and between states. Its deficiencies are pinpointed and conclusions reached as to changes needed. A modified program of areal stream gaging is described that would overcome some of the existing deficiencies. Progress made toward adapting such a program is reported and remaining problems are outlined.

INTRODUCTION

At least four thorough studies[2,3,4,5] have been made since 1949 to evaluate the adequacy of hydrologic data in this country. The strengths and weaknesses of existing data programs were analyzed at great length and a number of recommendations were made for needed improvements. Three of these studies

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

[a] Publication authorized by the Director, United States Geological Survey, Dept. of the Interior, Washington, D. C.

[b] Presented at the March 1960, ASCE Convention in New Orleans, La.

[1] Dist. Engr., U. S. Geological Survey, Trenton, N. J.

[2] "Adequacy of Hydrologic Data to Meet Federal Needs," Federal Inter-Agency River Basin Committee, 1949.

[3] "A Water Policy for the American People," President's Water Resources Policy Comm., U. S. Govt. Printing Office, Washington, D. C., 1950.

[4] "Water Policy," Presidential Advisory Committee on Water Resources Policy, U. S. Govt. Printing Office, Washington, D. C., 1955.

[5] "An Engineering Appraisal of Hydrologic Data," Task Group on Hydrologic Data of the Committee on Hydrology of the Hydr. Div., Proceedings, ASCE, Vol. 85, No. HY 7, July, 1959.

also attempted to estimate the required size of program or number of stations needed to provide adequate hydrologic data of the several principal types. The estimated number of stream-gaging stations needed, as determined in these three surveys, is shown on Fig. 1. Also plotted for comparison is the actual number of gaging stations, showing the growth rate since 1903.

It can be seen that all three of the surveys concluded that the number of gaging stations should be expanded rapidly. The surveys differed only slightly as to the total number of stations recommended by given target dates. Nevertheless, the growth rate of total stream-gaging stations slowed down about 1952 in the face of these carefully considered recommendations. To understand this paradox one must look at the existing program from several different perspectives and then follow the evolution of new concepts aimed at getting the maximum amount of hydrologic information for each dollar invested in stream gaging.

THE PROGRAM FROM SEVERAL PERSPECTIVES

Size and Cost of Program.—Operation costs for a stream-gaging station in the United States range from $600 per year to as much as $5,000 a year for a few remote stations, with an average of about $950 per station per year in 1960. Installation costs range from $1,000 to $15,000, and average about $3,000. The average life of a structure is on the order of 30 yr, making depreciation, without interest, about $100 per year.

The cost to the nation averages $950 for operation plus $100 for amortization of capital investment for each of about 7,150 stations, or a total of $7.5 million per year spent by the United States Geological Survey, Dept. of the Interior (USGS) alone. Of this, about one-third is contributed by the states on a 50-50 matching basis and nearly one-fourth is transferred to the USGS from other Federal agencies. Perhaps another million is spent annually by other Federal agencies within their own organization for stations closely related to internal operations.

The Mississippi River Commission annually publishes about 60 records of river stage and discharge and another 100 records of river stage only.[6] The International Boundary and Water Commission, United States and Mexico, publishes approximately fifty streamflow records collected in the Rio Grande basin.[7] The USGS also publishes some records collected by other agencies.

The total national investment, including state, county, and city expenditures made in cooperation with the USGS, is about $9 million each year for basic streamflow data. This amount represents less than 0.5% of the amount spent in the country annually for dams, levees, irrigation works, water supply sources, and other works dependent on streamflow data for safe and economical design.[8] Expressed another way, it costs about five cents per person per year for operation of streamflow stations at all levels of government. This expenditure is modest considering the extent to which man is dependent on water for luxuries, necessities, and indeed life itself. Yet continuing infla-

6 "Annual Report of Stages and Discharges of Mississippi River and its Tributaries," Mississippi River Comm., Vicksburg, Miss.

7 "Annual Report of International Boundary and Water Commission, United States and Mexico," El Paso, Texas.

8 "Water Facts for the Nation's Future," by W. B. Langbein and W. G. Hoyt, Ronald Press, N. Y., 1959.

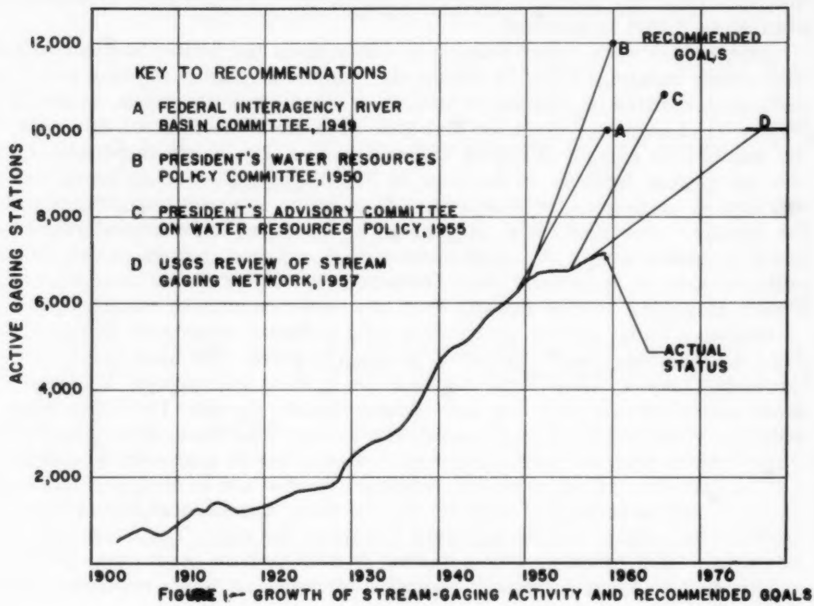

FIG. 1.—GROWTH OF STREAM-GAGING ACTIVITY AND RECOMMENDED GOALS

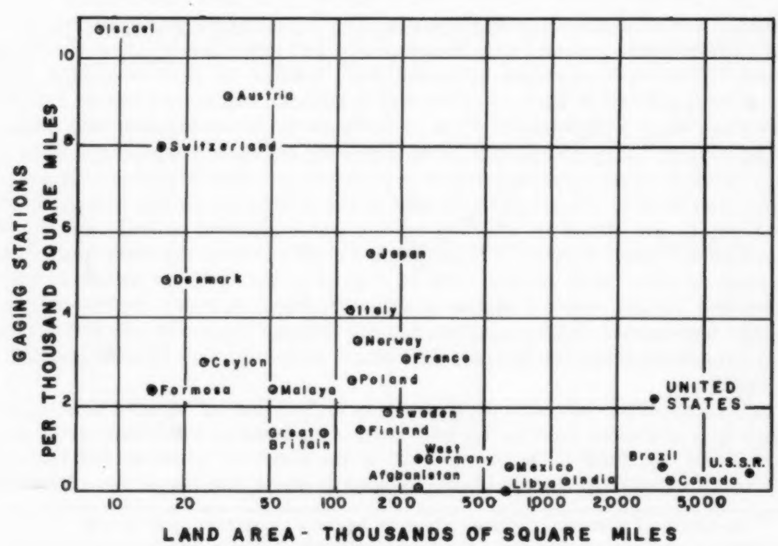

FIG. 2.—GAGING STATION DENSITY IN RELATION TO TOTAL AREA OF VARIOUS COUNTRIES

tionary trends and mounting Federal budgets make it imperative to economize even more if that is feasible.

Comparison with Other Countries.—How does the United States compare with other countries? Fig. 2 shows the number of gaging stations per thousand square miles in relation to total area for various countries, as listed by Walter B. Langbein.[9] It is evident that area alone is not a reliable guide to the number or density of gaging stations needed. The range is infinite. Libya has no gaging stations in an area of 633,000 sq miles, while Israel has 85 stations in an area of 8,000 sq miles. It might be inferred from this plot that the smaller countries have, of necessity, developed their natural resources more intensively than the larger countries, and that gaging-station density reflects this in a general way. Note that the United States is considerably higher in gaging station density than any other country of comparable size.

Gaging-station density in relation to population density is illustrated in Fig. 3. Including population as a parameter along with area makes the relationship take a vague form that can begin to be interpreted. Note that undeveloped countries plot low and usually toward the left. The correlation is still too scattered to be very useful. However, on the basis of population, the United States does not seem unexpectedly high or low in gaging-station density. As the intensity of development continues to rise and as the "population explosion" will presumably continue, the position of the United States might be expected to continue to rise and drift further to the right. The change in position since 1930 is shown by the dashed line. Of course, most other countries are tending to move in the same general direction at a rate dependent on the speed of development of their water resources.

Comparison of States Within the United States.—Perhaps better evidence that the number of gaging stations is related to population density can be seen in Fig. 4, as reported by Langbein.[9] This graph shows the relative position of each state in continental United States as of 1955, after empirical adjustment for irrigation and waterpower. The adjustments were determined by trial, their effectiveness being measured by reduction in scatter of the plotted points. Langbein's analysis indicated that, insofar as stream-gaging needs are concerned, each acre of irrigated farmland was equivalent to 4 people. Likewise, each hydro-power plant of 1,000 kw or more capacity was found to be equivalent to 10,000 people. With these adjustments the points tend to define a rather consistent relationship between number of gaging stations and population density. That is to say, that in the United States the stream-gaging program in all states is affected by the same national policies and that the standard of living or stage of economic development does not vary appreciably between states. New Jersey with 10.5 gaging stations per thousand square miles and Nevada with 0.5 station per thousand square miles represent nearly the two extremes in both population density (2nd and 48th) and industrialization in conterminous United States. All other states line up reasonably well on this plot.

Effect of Administrative Policies.—It is interesting to note that Alaska, which has replaced Nevada as our least developed and populous state since Langbein's analysis (1957) would plot off the sheet to the lower left but about in line with the other states. Alaska is the only state that has never cooperated

9 "Numbers of Stream-Gaging Stations in Various Countries with Analysis of their Distribution in the United States," by W. B. Langbein, Bulletin de l'Association International d'Hydrologie Scientifique, No. 8, December, 1957.

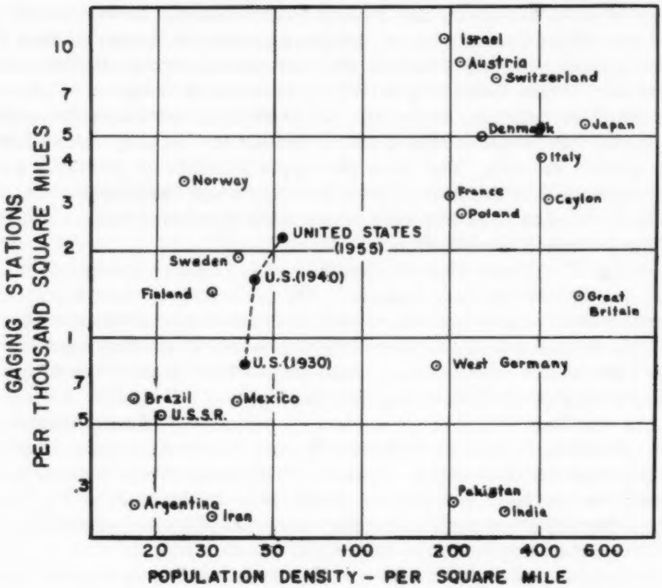

FIG. 3.—GAGING STATION DENSITY IN RELATION TO POPULATION DENSITY - 1955

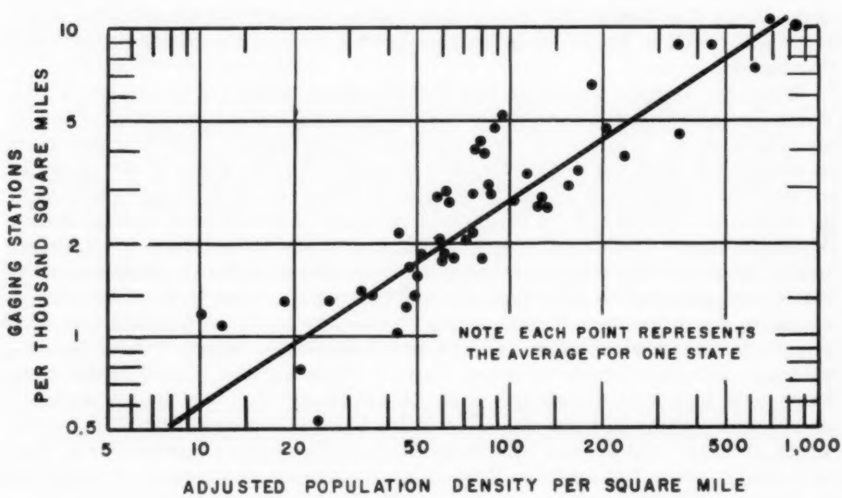

FIG. 4.—NUMBER OF GAGING STATIONS IN THE UNITED STATES IN RELATION TO POPULATION

on a 50-50 financial basis, and all gaging stations in Alaska prior to 1959 were financed completely by the Federal government. Hawaii would plot off the top of the sheet, having twice the gaging-station density of New Jersey, the previous leader in this respect, but only one-tenth the population density of New Jersey. While Hawaii is hardly typical, being made up of eight major and many smaller islands, it seems likely that the administrative policy of the Territorial government was a major factor in reaching such a high level in gaging-station density. The huge pineapple plantations probably exerted a major influence in this because of their irrigation and drainage needs. Hawaii, in contrast to Alaska, has for several decades furnished half of the stream-gaging dollars from local taxation.

In 1929 the Congress first authorized the Federal matching of state or territorial funds for stream gaging. This principle of 50-50 matching has been followed ever since and the Federal-state cooperation has become the backbone and major source of funds for the work. Referring again to Fig. 1, the sharp rise in the curve since 1929 shows very plainly the effect of this partnership policy on growth of stream-gaging.

Israel is another example of a country which, by administrative policy, decided it would have intensive stream gaging and ground water exploration. Taking advantage of all feasible means to "lift itself by the bootstraps" from a backward nation to a position of leadership in the Middle East, this tiny country has perhaps the most intensive and best planned hydrologic investigations' program of any nation in the world (as of 1960).

Preponderance on Large Streams.—The early pressures for gaging stations were for navigation, hydroelectric power, and flood control with the result that practically all older stations are on the larger streams. This had the advantage that approximately 80% to 90% of the runoff came under measurement rather quickly. The only surface runoff not being measured today in conterminous United States is in relatively small coastal streams and in tidal reaches of the larger rivers. In fact, some water originating in the upper Missouri or Ohio Rivers may be measured a dozen or more times before it reaches the ocean.

However, in most instances this flowing water would not be measured until it had passed through a run and a brook, or perhaps a creek or two, and finally entered a river draining several hundred square miles. Less than one-third of the gaging stations in the United States are on watersheds of 100 sq miles or smaller. Less than 5% of all gaging stations are on drainage areas of 10 sq miles or less despite a three-fold increase in small-area gaging-station coverage since 1945. Yet there are many, many more miles in aggregate length of small streams than there are total miles of the large streams. As the years pass, the smaller streams are becoming of ever greater economic, sanitary, and recreational importance. To be able to say with confidence what runoff is available in these smaller streams, or the extent of their flooding potential and other characteristics, more of them must be gaged. At the same time, new technical knowledge must be developed that will let the engineer or scientist interpolate the characteristics of the still ungaged small streams, which probably will always outnumber the gaged streams a hundred-fold. This is a large order because the flow characteristics become more diverse and variable as the drainage area becomes smaller.

It is interesting to ponder the question of the spacing of gaging stations. Assume the choice is one station for every 20, 50, 100, 200, 400, or 700 miles of stream. For small streams, a choice of one every 50 miles would be the same as saying there should be one on every tenth stream averaging 5 miles in length. Most engineers quizzed by the writer intuitively chose from 50 to 100 miles as a reasonable distance between stations, yet even the more conservative of these two choices would require quadrupling the size of the present network.

From spot sampling of stream density (map studies of perennial stream length divided by area) in various parts of the country, E. W. Coffay[10] has found the average to be slightly more than one mile length per square mile of area. Although this study has not been carried to completion (as of 1961), and the final answer may be slightly different, a rounded figure of 3 million miles of perennial streams in conterminous United States can be used safely for estimating purposes. Deducting stations in Alaska and Hawaii but including all other streamflow stations operated by the USGS and other Federal or state agencies (6680), the ratio comes out to one station now in existence for every 450 miles of streams.

There is no large stream in the country with gaging stations as far as 450 miles apart, on the average, so it must be concluded that stations are still farther apart on the small streams at present. In fact, stations now may average less than one for every 600 miles of aggregate length on the small streams. Admittedly, a mileage basis is no way to design a stream-gaging network; its only significance is to lend perspective as to the magnitude of the task of getting adequate data on all small streams.

Other Segments of the Hydrologic Cycle.—An excellent and up-to-date appraisal of the six principal types of hydrologic data was given in 1959[5] by the Task Group of Hydrologic Data of the Committee on Hydrology, Hydraulics Division, American Society of Civil Engineers (ASCE). It will serve here simply to quote excerpts of the Task Group's conclusions which are pertinent to this appraisal.

The Task Group found that:

> "The existing federal networks of gages measuring hydrologic data are producing satisfactory records. Periodic reappraisals are desirable for bringing about improvements to keep pace with changing needs and conditions. A reluctance to change prevailing practices can lead to over-collection and failure to begin records needed for the future."

The report noted several new types of data of current and future importance. These included natural or background radioactivity of waters and changes due to radioactive pollutants, data to evaluate effects of watershed treatment measures, and artificial rain-making.

In an earlier part of the Task Group report devoted specifically to runoff it was stated that:

> "The most obvious deficiency is in data from small drainage areas of 50 square miles or less. More data are needed on streamflow in the

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[10] Personal Communication with the author.

zone of tidal influence, and such data already collected should be assembled, analyzed and published."

INFERENCES AND TENTATIVE CONCLUSIONS

Looking at the national stream-gaging program from several different perspectives leads to the following tentative conclusions:

1. Present costs are not excessive; five cents per person per year or one-half of one percent of annual expenditures for water resources development.
2. The United States is not unexpectedly high or low in gaging-station density in relation to other countries. Considering the high standard of living and extent of industrialization in the United States, it may be lower than expected. On the other hand, the United States has considerably more stations than any other country of comparable size.
3. The need for streamflow data apparently is related to population density and extent of industrialization, both of which are projected to grow very rapidly in the United States in future decades.
4. Greatest present and future need is for data on thousands of small streams. Past methods of building permanent gaging stations wherever data are needed will not suffice even if funds and manpower could be doubled or tripled.
5. To keep pace with the widespread needs, the program must be directed toward obtaining less complete data but on a broader coverage—particularly on smaller streams and tidal streams. Also some new types of data should be observed and published to meet recently emerging requirements.

The phrase "areal stream gaging" is used to describe such a program, in which all available streamflow, flood, and drought records are evaluated along with topographic, geologic, and meteorologic information to determine regions or areas of hydrologic similarity. Within such an area the shape of the hydrograph is reasonably similar for streams of equal drainage area; peak runoff per square mile, average runoff per square mile, and base flow characteristics tend to be comparable. Within such an area only one or two complete gaging stations need be maintained continuously, with variations from place to place within the area being defined by reconnaissance methods.

REVIEW OF THE NETWORK

Development of Concepts.—The thinking and concepts previously expressed were actually the development of many years of careful consideration and study by many hydraulic engineers. The first person to document these ideas formally, as far as the writer is aware, was Walter Langbein in 1951. The pros and cons of these concepts were debated extensively within the USGS in the months that followed, and in 1952 a committee of six engineers was appointed to study this matter formally. This committee, of which Langbein was chairman, submitted its report in 1953. The report was reproduced and dis-

tributed within the USGS for study, comment, and field trial of the ideas and procedures proposed. The concepts of the committee were stated as follows:

> "The principles that are proposed as guides in the development of the gaging-station program in order to achieve maximum information relative to costs are the following: (1) Apart from operational needs, stream gaging is a sampling process, and (2) within limits it is possible to correlate the flow of one stream with that of another. A gaging station provides information on the rates and amounts of flow during a particular time in a particular channel and therefore samples a given time and given place. The variations that occur with respect to time at a gaging station are fairly well understood, but those that exist from place to place less so. A well-balanced network, therefore, gives the two factors of time and place their proper consideration. The two factors can be brought into balanced relationship through use of correlation techniques.
>
> "These basic principles lead to the conclusion that a record can be viewed not only as a measure of the flow at one site but also as an index to the flow of streams in the contigrous area—in effect magnifying its usefulness manyfold. These principles lead toward areal rather than spot stream gaging."

In November, 1955, Langbein and C. H. Hardison presented a paper[11] which described the theory, procedures, and statistics involved in the correlation of a short record with a nearby long-term gaging-station record. These techniques are very useful in applying the program concept of a hydrologic network of stations. The Presidential Advisory Committee on Water Resources Policy, in December, 1955, made recommendations[4] which added further impetus to this concept of a hydrologic network, made up of long-term and short-term stations. The USGS ran a pilot project in the Colorado River basin to test the feasibility of the concepts and the problems involved in attempting to adapt an existing stream-gaging program to these network principles. The problems were many but none insurmountable. The results of the pilot project were deemed to warrant adopting the network principles as a guide for future programming.

During the last half of 1956 and all of 1957, a formal and detailed review of the entire stream-gaging program in the United States and its territories was made by the USGS in accordance with these network principles. Since that time the USGS has been following these principles in its program design. Naturally, these concepts and the modification in program goals were discussed with representatives of other Federal agencies in the Subcommittee on Hydrology of the Inter-Agency Committee on Water Resources. They were also discussed with local cooperating officials in state and federal agencies at different times during the consideration, testing, and adaptation of these principles.

At no time during this period of more than 8 yr that these concepts have been developing have engineers of the USGS felt that they have known all the answers. Accordingly, there has been no great rush to get into print any final version of this type of network design. However, it is felt that the concepts have been sufficiently tested and that enough answers are known to warrant

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11 "Extending Streamflow Data," by W. B. Langbein and C. H. Hardison, Proceedings, ASCE, Vol. 81, Proceedings Separate No. 826, November, 1955.

publication. Furthermore, it has become apparent that lack of wide dissemination of the ideas and concepts has somewhat deterred their acceptance by the engineering profession and officials concerned with streamflow data programs.

Additional information has been presented by Langbein and W. G. Hoyt.[8] This book, sponsored by the Conservation Foundation, contains a comprehensive discussion of hydrologic data networks and programs. Therefore, only general principles, procedures, and results will be mentioned here.

TABLE 1.—CLASSIFICATION OF SURFACE WATER RECORDS

Streamflow Station	Stage Station	Partial Record Station
Hydrologic Network	Hydrologic Network	Hydrologic Network
Primary (long term)	Primary (long term)	Primary (long term)
A 11 Areal (including necessary supporting records	B 11 Areal (Ponds & lakes)	None
A 12 Mainstream	B 12 Mainstream (Rivers & tidal estuaries	
Secondary (short term)	Secondary (short term)	Secondary (short term
A 21 Areal	B 21 Areal (Ponds & lakes)	C 21 Flood crest
A 22 Mainstream	B 22 Mainstream (Rivers & tidal estuaries)	C 22 Low flow
		C 23 Pond & lake inventory
A 23 Seasonal daily - hydrologic		C 24 Periodic streamflow
Water Management	Water Management	Water Management
Long term	Long term	Long term
A 31 Compact	B 31 Compact	
A 32 Legal	B 32 Legal	
A 33 Operational	B 33 Operational	
A 34 Administrative	B 34 Administrative	
A 35 Basin Accounting	B 35 Basin Accounting	
A 36 Federal Power Comm.	B 36 Federal Power Comm.	
Short term	Short term	Short term
A 41 Research & experimentation	B 41 Research & experimentation	C 41 Flood hydrographs nad and timing
A 42 Detailed design	B 42 Detailed design	
A 43 Operational (including seasonal)	B 43 Operational	C 42 Seepage and low flow

Classification of Stations.—The first step in the actual modification of the program was to review and classify all existing gaging stations as to their best use in a nationwide network. Table 1 shows the classification system adopted. Each main category was further divided into hydrologic network or water management stations. A water management station is defined as one needed to provide data for one or more specific operational, legal, or admini-

strative purposes. Its establishment, location, and length of operation depend almost entirely upon the needs of the project or projects for which it is operated. Accordingly, it was decided at the outset that this type of station could not be programmed in advance by consideration of general basin hydrology and general purpose needs. The programming problem was thereby narrowed down immediately to the design of a general purpose hydrologic network. However, in all cases for which a gaging station was found in the original review to serve hydrologic network purposes as well as certain water management purposes, the station was classified as a hydrologic network station for economy reasons.

The hydrologic network was broken down into primary stations and secondary stations. The primary stations are the long-term stations to be operated for an indefinite period of time. Secondary stations are intended to be of short-term operation, say from 5 yr to 10 yr. At the end of that time the station, and as much of the equipment as can be moved, would be relocated to some other point needing stream gaging. Both the primary and secondary classifications were further broken down into areal and mainstream stations.

Primary Stations.—Areal primary stations are index stations on streams that must essentially be free from past regulation and, hopefully, free from extensive future regulation, diversion, or other development. In parts of the United States it is becoming increasingly difficult to find stations completely free from regulation and diversion. This made the job more difficult but nowhere prevented the use of the network principles. Thus an areal primary station is selected for representativeness and length of record; insofar as possible it is free from past and future regulation or diversion, and will be operated for an indefinite period of time to obtain a long range time sample of the hydrology of the section in which it is located. This station record would be used as the independent variable in making correlative estimates of long-term streamflow characteristics at other sites in the same hydrologic province.

A mainstream primary station, on the other hand, may be considerably affected by regulation or diversion. It serves only as as a record of actual flow at that point and as an index of flow at other points upstream or downstream on the same large river. An example of this would be the Mississippi River at Vicksburg, at which point the river flow would correlate very closely with simultaneous flow in the river at other points upstream or downstream for many miles, but would not bear any particular relationship to the flow in small tributaries entering the Mississippi River in this vicinity.

The areal primary stations selected were those which seemed to be most representative of the hydrologic characteristics of the area in which they were located. Trial correlations of monthly mean flows for a 5-yr period, generally 1951-55, were used for this purpose. It was found that stations with either very large or very small drainage areas did not correlate well with surrounding stations. Therefore, most areal primary stations, as a result, are stations with medium size drainage area. Attention was given to this point to insure that some primary stations were selected on smaller basins and some on larger drainage areas in all parts of the country in order to avoid complete bias in the final network.

Secondary Stations.—An areal secondary station would be operated where general streamflow information is wanted or will likely be needed in the future, the length of the required record depending on the number of years

necessary to define a correlation with a nearby areal primary station adequately. A mainstream secondary station, likewise, would be operated for only as many years as needed to define a correlation with a nearby mainstream primary station. Then either type of secondary station might be discontinued or converted to a partial-record station. Because of their frequent relocation, the design of new secondary stations will feature prefabricated, demountable, and semi-portable type shelters that can be quickly knocked down and moved.

Partial-Record Stations.—The principal emphasis on partial-records is on flood peak flows and base flows, the two extremes of river regimen. Flood peak partial-records are frequently obtained by use of the "crest-stage gage." A small vertical pipe on a tree or bridge abutment has inlet holes near the bottom and is charged with a spoonful of granulated cork. After a flood the cork leaves a high-water mark on a graduated stick left in the pipe. The engineer visits the station periodically, preferably after each large rise, and notes the peak stage since the previous visit, removes the mark, and re-charges with cork. The cost of operating such a station is from 5% to 30% of a complete gaging station cost, depending on need for and difficulty of calibration of the site to develop a rating curve.

Base flow measurements are made several days to a week or more after the most recent rain on an area. The flow includes no direct storm runoff and is essentially all from ground-water discharge. Base flow is systematically measured in the spring, when the ground-water table is ordinarily at its maximum elevation for the year, and then measured again in the late summer or fall, when ground-water levels and discharges to the streams are usually at or near the minimum for the year. Thus, two widely separated points on the correlation graph are obtained each year for several years to define the base flow at any particular site in comparison to a nearby secondary or primary gaging station. The cost is about 5% to 10% of the cost of a complete gaging station.

Streamflow Correlations.—The correlation technique used in the nationwide review of the stream-gaging network was the comparison of simultaneous monthly mean flows. A graphical solution was usually made, as shown in Fig. 5. At least 60 months (5 yr) were plotted and a smooth mean curve drawn through these points. The standard error was estimated by drawing two curves parallel to the mean curve at such distances from it as to exclude the top one-sixth and bottom one-sixth of the plotted points or months, thus including two-thirds of the points, or the part included within plus and minus one standard deviation. In this example, two-thirds of the months were between +22% and -18% of the mean curve.

In general, the correlations of streamflow characteristics were found to be good in the humid east and southeast. Correlations were good to fair in the Midwest and northwestern parts of the country, and fair to poor in the arid southwest and states immediately east of the Rocky Mountains.

Other possibilities for making correlations between primary and secondary stations or partial-record stations are available. Concurrent daily flow, during base flow conditions, is especially useful for low flow partial-record stations. Discharge for equal percent duration or equal recurrence interval could also be used instead of monthly mean flows to extent records at a short-term secondary station. The extremes of flow at the short term station may also be estimated by a relation developed with the long term station but

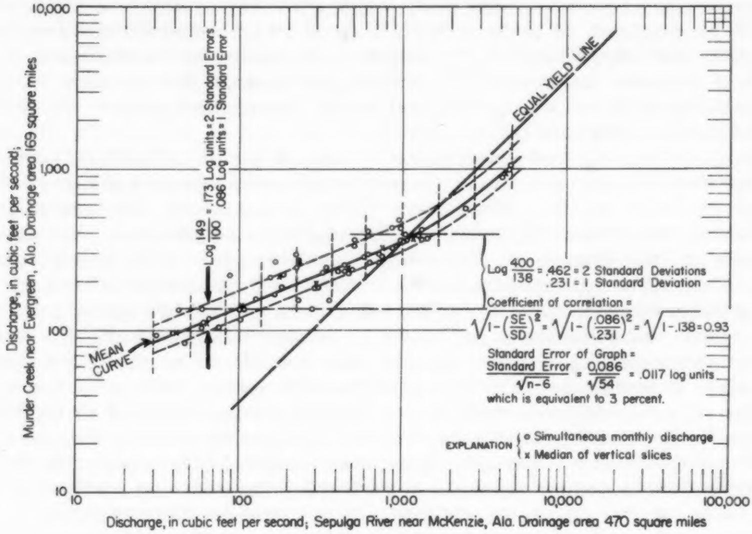

FIG. 5.—CORRELATION OF SIMULTANEOUS MONTHLY MEAN FLOWS

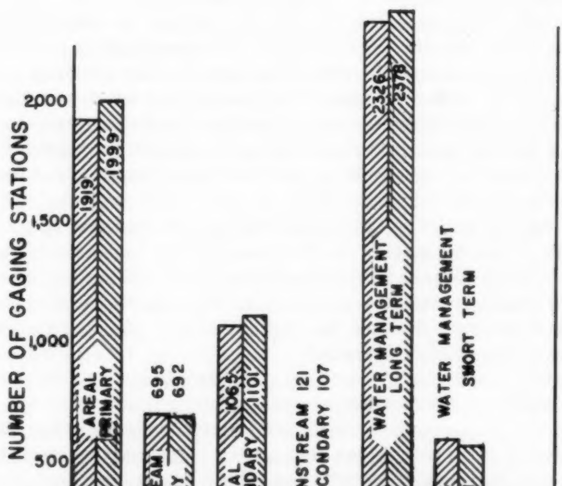

FIG. 6.—STREAMFLOW STATIONS IN EACH CLASSIFICATION AND RECENT TRENDS

it is usually preferable, especially where great extrapolation of the relation would be required, to make a study of flood frequency or drought frequency by other methods. There is the additional prospect that rainfall data can be used to improve the correlation between the flows of two streams. Multiple correlation using records for several nearby gaging stations may sometimes, but not always, improve the results.[12]

Scope of the Review.—Insofar as it was practicable, the USGS' review of the network of gaging stations included stations operated by states or other agencies such as the Mississippi River Commission, the International Boundary and Water Commission, United States and Mexico, the Corps of Engineers, the Bureau of Reclamation, and agencies of the Department of Agriculture. The criteria for inclusion of stations operated by other agencies or states required that the station records be published regularly and that the station be suitable for inclusion in the hydrologic network. If a station record collected by another agency was considered accurate and readily available to potential users, it was felt that there would be no need for duplication with an additional hydrologic network station operated by the USGS. Stations, operated by other agencies, that would be of use for water management purposes only, however, were not classified. These policies yielded coverage that is somewhat less than perfect for the nation as a whole. A considerable number of gaging stations are operated by other agencies in connection with research or operational projects, but the basic data have not been published though the evaluated results may have been included in special reports. Nevertheless, probably in excess of 95% of all existing stations in the country were included in the review. So the results are considered valid and useful from the standpoint of national coverage.

Results of the Review.—Certain aspects of the review were quite direct and yielded definite results. For example, it was possible to assign a classification of best use in the nationwide program to every existing gaging station. As might have been expected, even this was not timeless and there have been a number of reclassifications since the initial review was completed. However, the number of reclassifications has been very small compared to the number of stations, less than 1% in 3 yr, so the classification of existing stations may be considered fairly firm.

Another firm finding was that some 700 specific basins in this country did not have a single gaging station within the area that met areal primary station requirements. Most of these basins are in the Great Plains and Rocky Mountain areas where existing station density is low and variability of hydrologic characteristics greater than in the humid areas. The correction of this deficiency is being given high priority.

The review showed that more than 1,200 existing secondary stations could be moved to other locations after a period of operation of 5 yr to 10 yr, and that a good share of these were ready for immediate relocation. It was further shown by brief correlation tests that about 1,150 gaging stations previously discontinued had accumulated sufficient length of good records to permit adequate correlation with primary stations. It was also found that some 1,250 existing stations classified in the water management categories would qualify for the secondary hydrologic network (except for relocation after 10 yr). Thus, a total of about 3,600 points have been or are being gaged in the secondary

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[12] "Graphical Correlation of Gaging-Station Records," by J. K. Searcy, U. S. Geol. Survey, Water-Supply Paper 1541-C, 1960.

hydrologic network even though only 1,200 stations are currently classified as secondary stations.

A result, much less firm than the preceding ones, was the tentative conclusion that about 2,500 additional short-term secondary stations would be needed in the next ten years. This conclusion was reached by adding up the specific sites which, in the judgment of the 44 district engineers of the USGS across the country, would be needed in their districts. Thus, it is strictly a matter of judgment and cannot be proved right or wrong although it is the best guide available.

The last, and perhaps most hypothetical, result of the review was the indication that a total of about 10,000 active gaging stations in the United States is the maximum number that would ever be needed and justifiable at any one time. This figure was arrived at by adding the 700 additional primary and 2,500 additional secondary stations needed, as previously described, to the existing network and assuming that for every station established thereafter another one would be dropped. The figure of 10,000 assumes a slow but steady increase in water management stations and a very large increase in partial-record stations for reconnaissance type data. No specific date was set as a target for obtaining 10,000 stations. A straight line projection on Fig. 1 indicates reaching this level about 1975-85, much later than the goals established in the three previous surveys.

A side-effect, if not a result, of the review was the strengthening of the conviction of those participating in the study that a broadening of the types of data collected would be more valuable than simply adding more years of daily discharge record at all existing gaging stations. Data on flood plain inundation, water use, time of travel, flood profiles, bankful stage, water temperature, and chemical quality (to name only a few), are becoming more important and necessary every passing year.

Trends Since the Review.—Fig. 6 shows the number of streamflow stations in each classification as of January, 1957, and July, 1959. Of the 700 additional areal primary stations the review found to be needed, about 80 had been established by July, 1959. Only a net of 36 additional areal secondary stations were in operation but nearly 350 new secondary stations had been established during the period. Similarly, for all classifications of streamflow stations, data were being collected at 660 new locations, but because of selective discontinuation the net increase was only 115 stations.

There were very few changes in number of stage stations during this period, 34 being established and 17 discontinued for a net increase of 17. However, the expansion in the number of partial-record stations during the period was particularly encouraging. The exact number in existence in 1957 is not known but is believed to have been about 500 low-flow and 800 crest-stage stations. The count in September, 1959, shows growth to more than 2,100 low-flow stations and about 1,950 crest-stage gages.

At the risk of misleading the casual reader, the increases in the several categories and classifications can be totalled, though strictly speaking each is different from all others. It can be generalized that surface-water data are being collected at nearly 3,500 new points since January, 1957; the data are less detailed and require analysis for full use, but the cost in funds and manpower has risen only about the equivalent of 600 new complete gaging stations.

Remaining Problems.—Although the concept of a hydrologic network seems to be working out very well, not only in theory but in actual practice, there are quite a few remaining problems. The most immediate, if not the largest, of these problems is that of gaining acceptance of this type of program design by engineers and hydrologists throughout the country. So far there has been no outright disagreement with the concepts or opposition to the policies from anyone close enough to the problem to study it through. However, the ideas and the procedures have not been well advertised and thoroughly discussed in the literature as yet, to inform fully all who need to know about this approach. Natural resistance to change until the reason for change is fully understood has arisen in a few isolated instances. These can probably be regarded as the exceptions that prove the rule. Most of these instances arise when it is proposed to discontinue a secondary station which is no longer needed for hydrologic network purposes. Better understanding of concepts and program goals will gradually overcome this problem.

Another fairly urgent problem appears to be the acquisition of funds to establish "bench mark" areal primary stations found to be needed. A "bench mark" station is one in a national or a state park, a wilderness area, or other area where hydrologic change due to man's activity does not appear likely. There are few existing stations where the hydrology of the basin will not change perceptibly in the next one hundred or more years. Wherever a basin can be found in which long-term, man-made change in the hydrologic regimen seems unlikely, an areal primary gaging station should be established to detect and document long-term trends as between wet and dry cycles. If an average of two or three such stations could be found in each state, they would represent a wonderful long-term investment in better understanding of regional hydrology and more reliable determination of the probable frequency of recurrence of given floods or droughts.

Another problem arising as a result of this type of programming is getting all basic data assembled and published for wide use. Data from partial-record stations and other reconnaissance-type information must be either fully evaluated and interpreted in published reports or the partial-record data and reconnaissance data themselves must be published. It is planned to publish data from crest-stage gages and base-flow stations along with continuous station records in water-supply papers of the USGS.

The final problem which needs mention is that of getting underway more special studies and analyses in which all available hydrologic data are assembled into interpretive reports. These reports should contain maps, graphs, and tables to make the great mass of accumulated hydrologic data more readily useful to practicing engineers and other data users. In addition, shorter and less technical reports should be prepared directed toward school children and general public readers. These should contain the more significant facts to promote wise public use, control, and conservation of the nation's water resources. The investment in data collection will be partly wasted if the knowledge gained is not fully disseminated.

SUMMARY

The stream-gaging program was reviewed from several different angles and found to be generally adequate. To overcome existing deficiencies on

small streams, and to orient the program better toward future needs, certain changes were found to be desirable. Accordingly, a flexible system has been adopted with a nearly permanent primary network of index stations and a continuously changing secondary network of short-term stations. Both networks are supplemented by a large number of partial-record stations designed to give specific data on flood peaks or low flow.

The whole system is mutually supporting and correlations between primary, secondary, and partial-record stations will make data collected at each much more useful and valuable than if used alone. The network system provides a logical guide for efficient expansion of the program and, if need be, for reductions with minimum damage. Above all, the program is designed to speed up greatly reconnaissance-type coverage on small streams and to give maximum information for the manpower and funds expended. The growth rate, in terms of total stream-gaging stations being operated, has slowed down as a result of network concepts. However the growth rate, in terms of total significant streamflow data being collected, has speeded up markedly.

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STILLING BASIN DAMAGE AT CHIEF JOSEPH DAM[a]

By R. H. Gedney,[1] M. ASCE

SYNOPSIS

Chief Joseph Dam on the Columbia River, in the state of Washington, is the first dam downstream of Grand Coulee Dam. The dam, approximately 200 ft high, is a concrete gravity structure with a tainter-gate controlled ogee spillway, that discharges into a hydraulic jump-type, energy dissipator having a single row of baffles and an end sill. Damage of the concrete in the basin was discovered in 1957, two years after the project became operative. The results of condition surveys made in 1957 and in 1960 are presented together with an evaluation of conditions that are considered to have contributed to the stilling basin damage.

INTRODUCTION

The occurrence of damage to spillway energy dissipators is not an uncommon event. However, there is seldom an opportunity to determine whether the events causing this damage are attributable to construction or to operating conditions and to define the specific situations responsible. The repair of a spillway energy dissipating structure for a major dam after a project is placed in service is often costly. Therefore, a full understanding

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

[a] Presented at Seattle, Washington Meeting, Hydraulic Division Conference, August 18, 1960.

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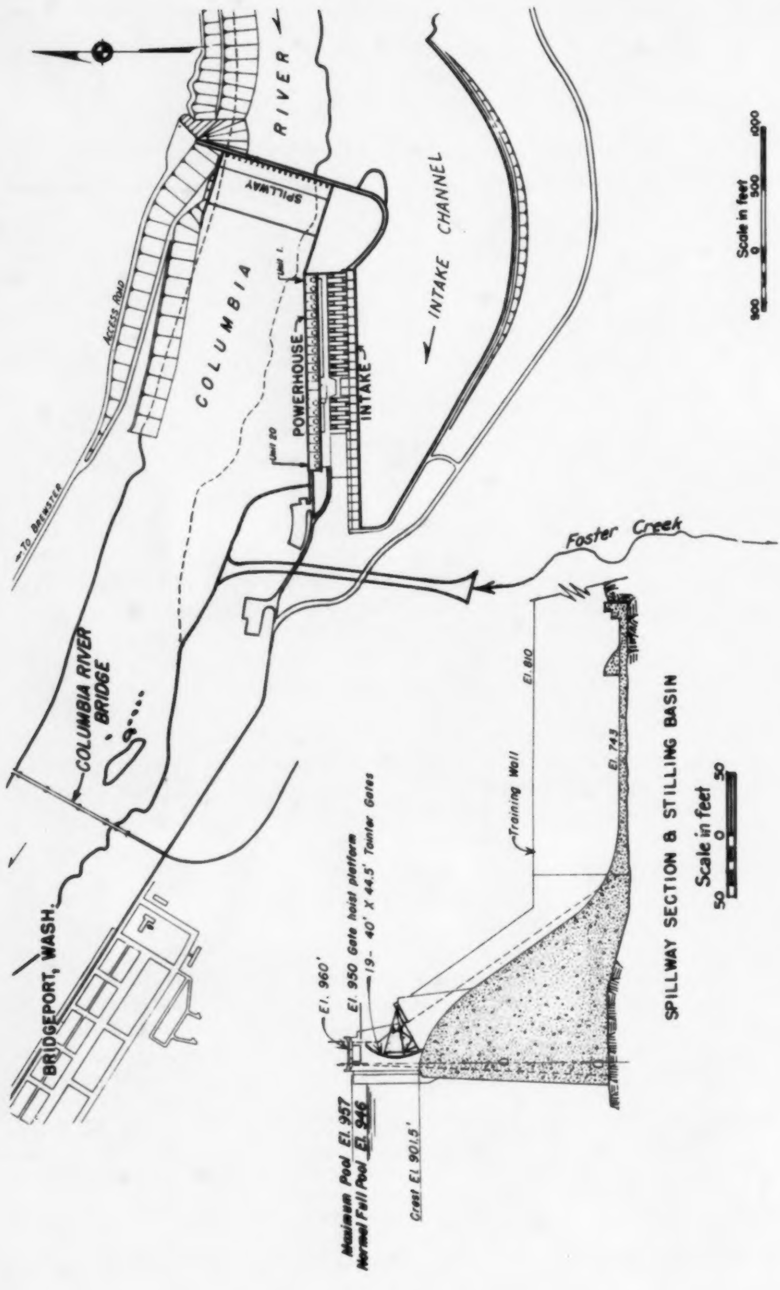


FIG. 1.—PLAN OF DAM AND POWERHOUSE

of circumstances leading to the occurrences of damage in Chief Joseph Dam stilling basin, that is a conventional structure constructed in accordance with conventional procedures, should be helpful. It is interesting to note that damage to spillway energy dissipators in varying degrees has occurred at Grand Coulee, Chief Joseph, Rocky Reach, and Bonneville Dams, all of which are major structures on the Columbia River.

PROJECT LOCATION AND SPILLWAY FEATURES

Chief Joseph Dam is located 51 miles downstream from Grand Coulee Dam and backs water into the Coulee Dam tailrace. The dam primarily consists of a concrete gravity gate-controlled spillway spanning the river channel, and an intake structure and powerhouse located just downstream, as shown on Figs. 1 and 2. Use of the spillway is required annually, generally over a 5- to 7-month period during which time peak flows of about 350,000 cfs to 500,000 cfs can be expected. The historical flood on the Columbia River at Chief Joseph site is about 730,000 cfs. The flood of record occurring in 1948 is 640,000 cfs, and the spillway design discharge is 1,250,000 cfs.

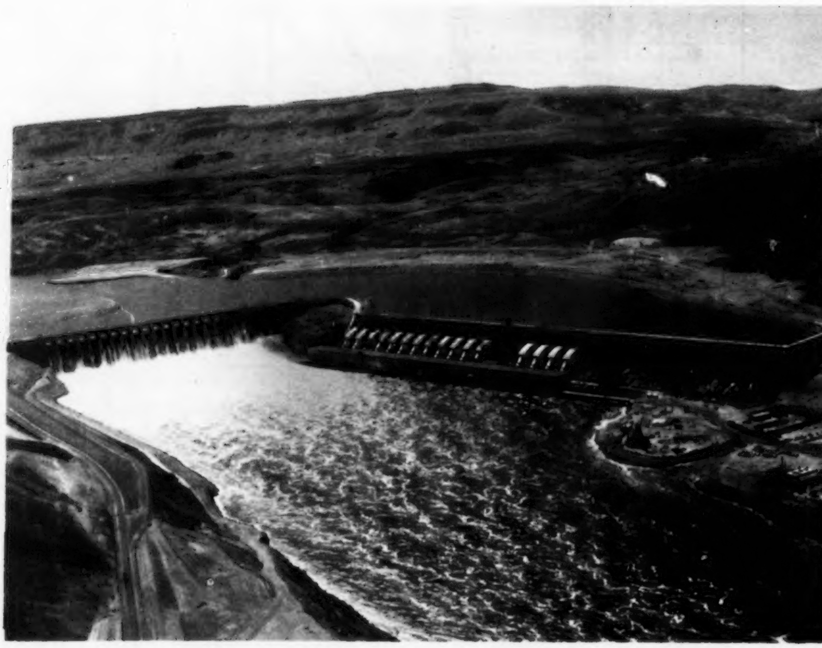


FIG. 2.—CHIEF JOSEPH DAM. FLOW OVER COMPLETED SPILLWAY.

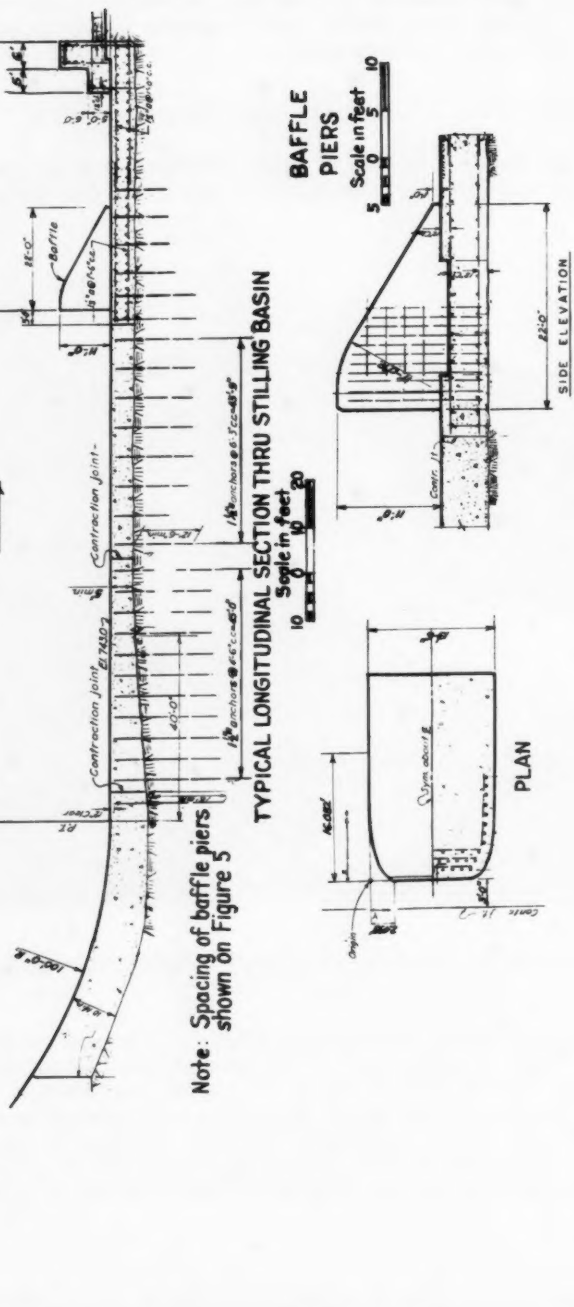


FIG. 3.—STILLING BASIN DETAILS

Normal head at the project during ordinary flood periods is in the order of magnitude of 150 ft from pool to tailwater and 200 ft from pool to stilling basin floor.

Flow over the spillway is controlled by nineteen tainter gates, 44.5 ft high and 40 ft wide. During flood periods, the pool will ordinarily be maintained at Ele. 946 to 948 with a maximum rise to Ele. 957 for the spillway design flow. A paved concrete stilling basin that induces a hydraulic jump for energy dissipation is provided at the toe of the spillway. The stilling basin is connected to the ogee by a 100-ft-radius bucket section and is 167 ft long, from point of tangency of the bucket to the end sill, as shown on Fig. 3. A single row of baffles and a stepped end sill are located in the downstream portion of the stilling basin. The stilling basin details and general plan are shown on Figs. 3 and 5. The stilling basin floor is comprised of slabs designated in Fig. 5 from Row D upstream to Row A downstream.

TABLE 1.—HYDRAULIC JUMP DEPTHS

Flow designation (1)	Flow Q in cfs (2)	Q per feet cfs (3)	D_1 ft (4)	V_1, in ft per sec (5)	D_2 ft (6)	D_2'/D_2 (7)
Spillway[a]	1,250,000	1,360.	12.0	114.	91.0	0.90
Historical maximum	730,000	800.	7.3	110.	70.6	0.92
3-Year Frequency	400,000	440.	3.9	113.	53.7	0.95

[a] Pool at Elev. 957, all other pools at Elev. 946.

D_1 = Theoretical depth at toe of jump.

V_1 = Theoretical velocity at toe of jump.

D_2 = Theoretical required tailwater depth to form jump.

D_2' = Actual tailwater depth available.

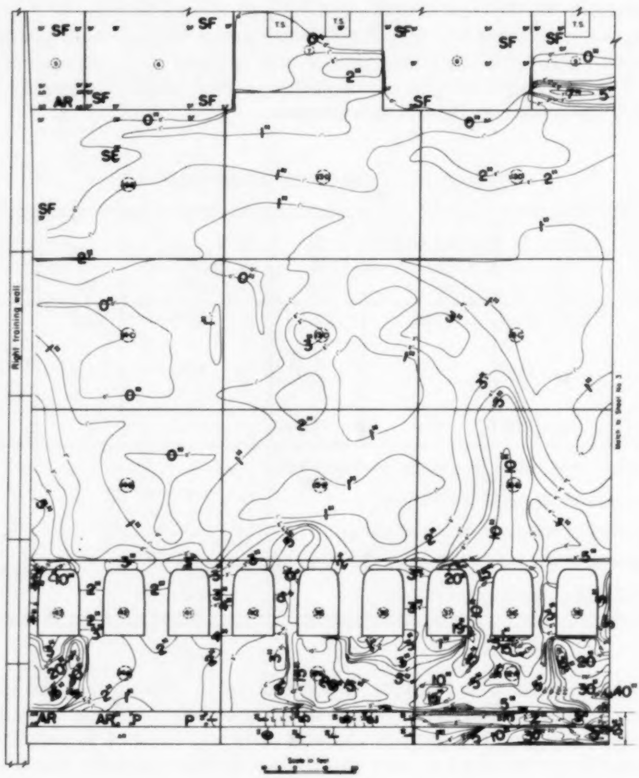
The slabs have a minimum thickness of 5 ft and are tied to the rock foundation with an extensive system of grouted anchor bars. Only the downstream slab on which the baffles and end sills are located is reinforced; this slab is heavily reinforced to take the impact of high velocity flows on baffles and end sill. The floor of the basin is level at Ele. 743, about 12 ft below streambed. An extensively jointed granitic rock underlies the basin and is found at or near the surface of the streambed downstream of the stilling basin. The rock jointing varies greatly in strength and in many areas the rock is subject to erosion under high velocity flow.

The stilling basin causes energy dissipation by hydraulic jump action. A single row of baffle piers, streamlined on the sides to reduce cavitation potential and a stepped end sill cause the jump to occur on the basin under maximum flow conditions and also aid materially in damping downstream wave action. The jump conditions in the basin for spillway design, the historical maximum flood and a 400,000-cfs flow (3-yr frequency), are given in Table 1.

If allowance is made for the portion of the river flow that will be diverted through the powerhouse, the D_2'/D_2 ratio would be in excess of 1.0 for flows

experienced as of 1960. The design features of the stilling basin were thoroughly tested in model studies, using a 1:33 scale, 3-gate sectional model, and a 1:80 scale general model that included the entire spillway.

One of the outstanding features of the design is the coordinated effect of the baffle piers and end sill in causing formation of the jump. The baffle piers were located close enough to the end sill so that the end sill aids in creating a pressure rise in the vicinity of the baffles that minimizes the occurrence of



(a) SHEET 4

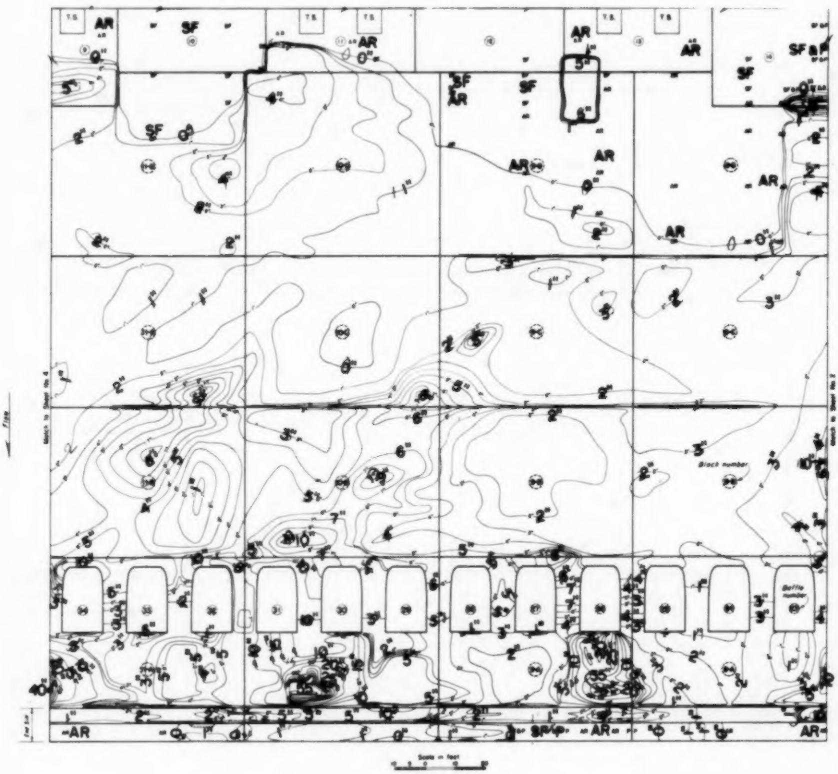
FIG. 4.—STILLING BASIN EROSION CONTOURS

negative pressures. The basin was carefully tested in a 1:36 sectional model to obtain an adequate length and depth and to avoid cavitation pressures on baffles and end sills. The model tests indicated an adequate margin of safety for satisfactory stilling basin performance for all flows and showed no indication of cavitation pressures for flows of less than 900,000 sec-ft. A significant feature disclosed by the model tests was that a slight variation in the curve of the sides of the baffle piers from the theoretical shape specified,

resulted in high negative pressures at design flow not present when the exact shape was used.

INITIAL REPORT OF DAMAGE TO SPILLWAY

Water was first diverted over a completed section of stilling basin in 1952. Thereafter, until the project was placed in service in 1955, a portion



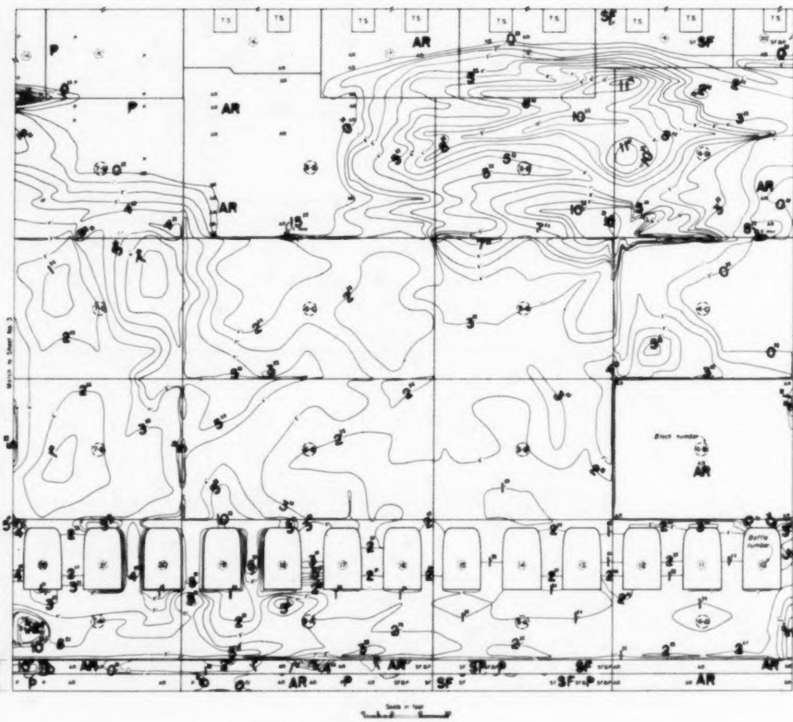
(b) SHEET 3

FIG. 4.—*CONTINUED*

of the stilling basin was utilized continuously each year under construction conditions. Continuous use of the spillway lasted until early in 1957 when sufficient hydraulic capacity was installed in the powerhouse to permit complete diversion of low water flows. Thus, initial inspection of the basin was delayed until March 1957.

The first inspection of the basin, made over a two-day period, consisted primarily of four underwater traverses by a diver from one side of the basin

to the other covering the end sill, the baffle piers and the paving from end sill to the spillway ogee section. This inspection revealed several large areas in which erosion had occurred to depths of zero to 12 in.; several extensive areas in which erosion exceeding 12 in. had occurred and in which there was exposed reinforcing steel and marked damage to a number of baffles and several sections of end sill. The surface of the slab was eroded



(c) SHEET 2

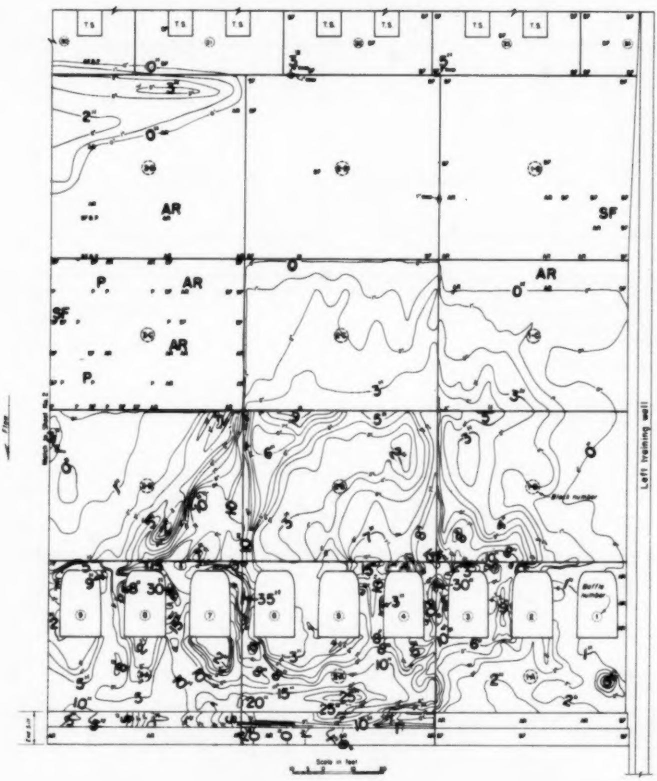
FIG. 4.—*CONTINUED*

in many areas to expose the 6-in. maximum aggregate used in spillway paving. Although no evidence of rock from construction operations was found in the basin. There were numerous loose pieces of aggregate in the basin that had been rounded and reduced in size to 2-in. or 3-in.-diameter spheres by washing action. Several loose pieces of broken 1-1/2-in. square reinforcing bars were also found in the basin. Eight or ten baffles were reported as

severely damaged as were localized sections of end sill. About half the baffles were reported to have sustained negligible or minor damage.

RESULTS OF SURVEY OF NOVEMBER 1957

A detailed underwater survey of the basin was undertaken in November, 1957. A team of two divers using specially fabricated equipment measured



(d) SHEET 1

FIG. 4.—*CONTINUED*

elevations over the floor of the basin on a grid spaced at approximately 10-ft centers and also made detailed measurements of baffle piers and end sill erosion.

The results of this survey are presented in detail in Fig. 4 and condensed to show overall major damage areas on Fig. 5. Fig. 4 indicates the erosion contours with the amount of erosion indicated in inches. The symbols indi-

cated in Fig. 4 are defined as follows: SF indicated a sand finish, with no appreciable erosion of concrete; P is pea gravel in relief; AR defines aggregate in relief, fines eroded from between coarse aggregate, and coarse aggre-

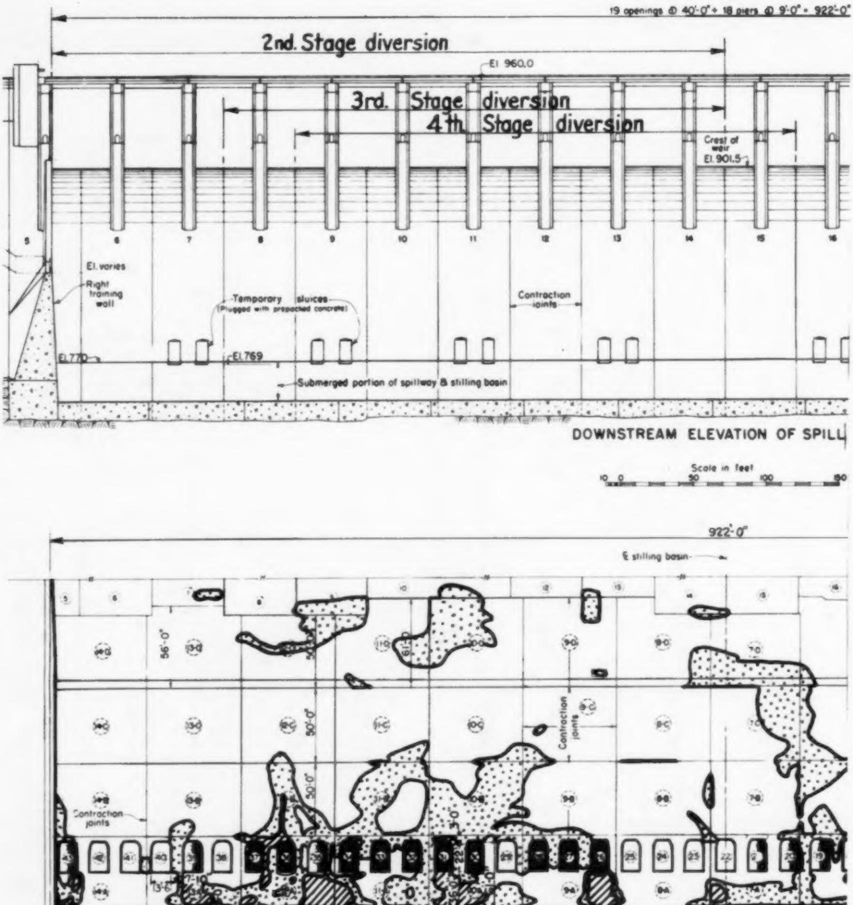
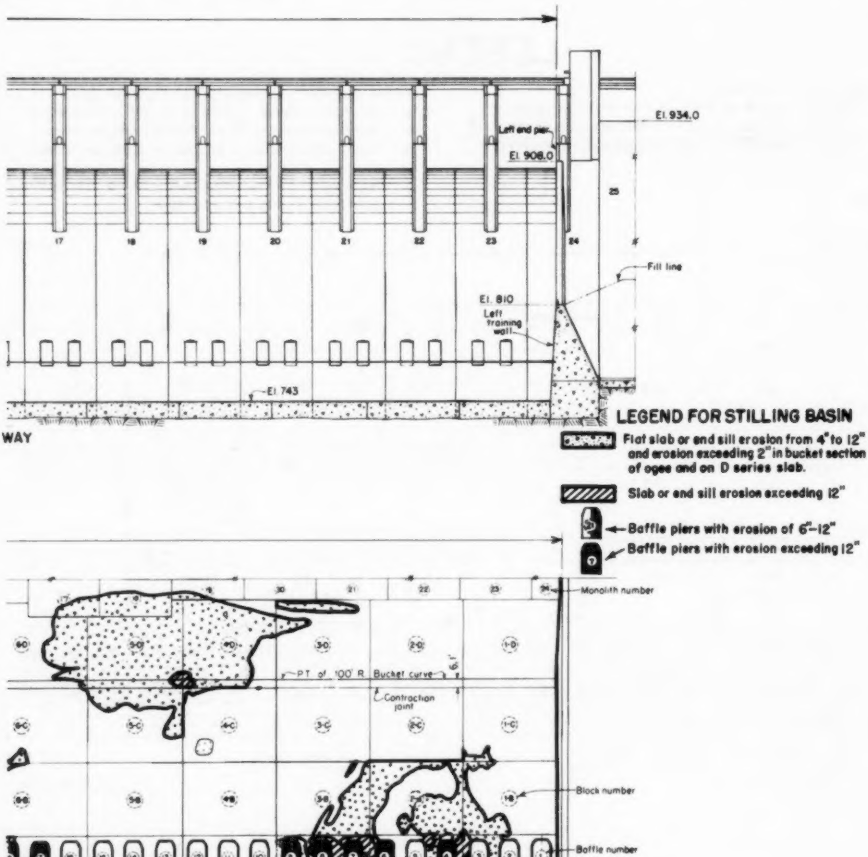


FIG. 5.—STILLING BASIN

gate not eroded: T.S. denotes temporary sluice, that is presently (1960) filled with concrete. Scarcely a single slab within the basin failed to show some degree of erosion. Because the aggregate used in paving of the slab was

graded up to a 6 in. maximum size, any erosion of the slab concrete results in development of an extremely rough surface as indicated in Fig. 12, and as reported by divers. Major damage areas of sizeable proportions with



EROSION, 1957 SURVEY

erosion exceeding 12 in. and more lightly eroded areas with erosion of less than 12 in. are indicated on Fig. 5 by various types of hatching.

Fig. 5 shows that only in the "A" row of slabs did erosion exceed 12 in. The most severe damage appears concentrated in localized areas immediately in front of baffle piers 37 to 26, and 3 to 9, between these baffles and between the baffle piers and end sill. The areas of greatest slab damage appear to

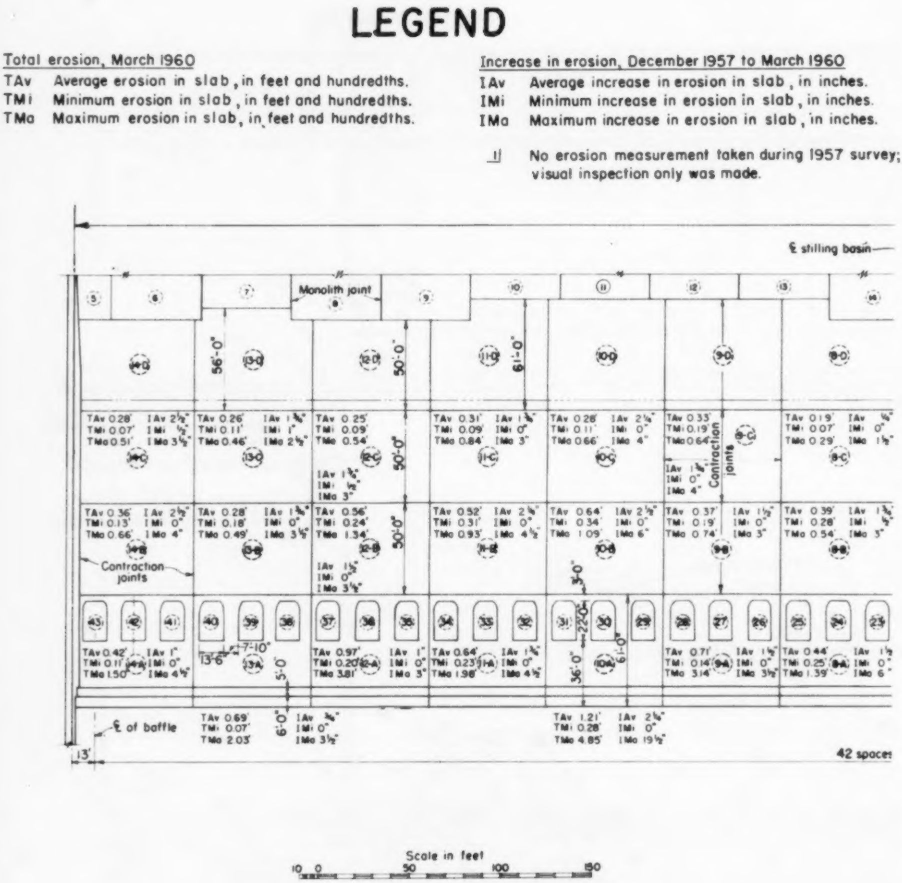
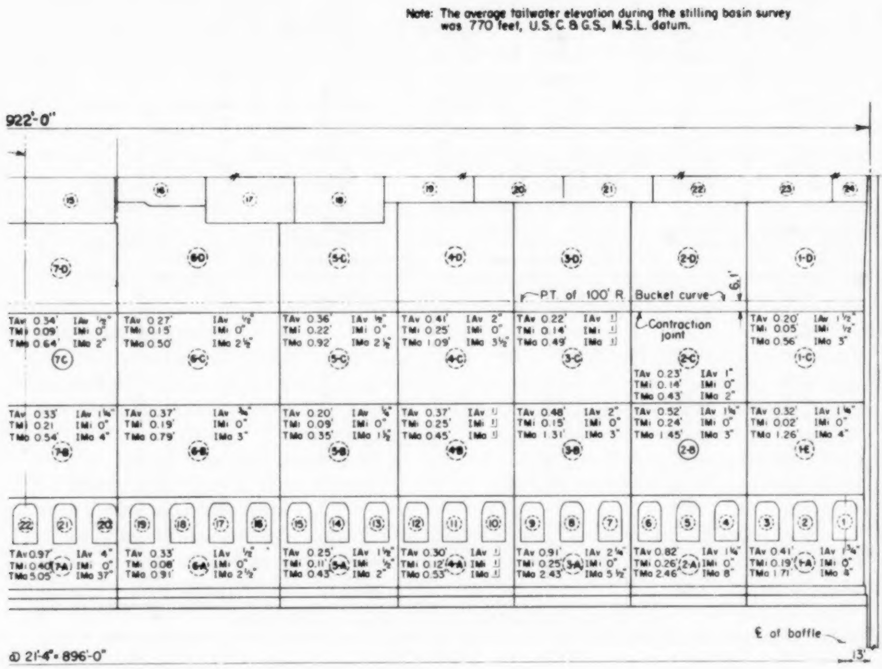


FIG. 6.—STILLING BASIN

coincide with locations in which baffle piers and the end sill also suffered severe damage. Two small scour holes approximately 5 ft deep were found downstream of baffle piers 22 and 23, and 30 and 31; other areas in which deep erosion occurred were generally less than 2 ft in depth. In the "A"

row of slabs, that is the only row with reinforcement, the surface layer of steel is extensively exposed in locations at which exceeds 12 in.

Detailed measurements were made and plotted of damage to each baffle. Of the forty-three baffles, it was found that twenty showed erosion of 6 in. or



EROSION, 1960 SURVEY

less and could be considered in satisfactory condition; seven showed erosion of 6 in. to 12 in., indicating near exposure of reinforcing steel; and sixteen showed erosion exceeding 12 in. that exposed reinforcing steel. The last two categories of baffles are indicated by shading on Fig. 5. Damages to baffles

TABLE 2.—PAVING EROSION DATA CHIEF JOSEPH DAM STILLING BASIN

Slab No.	Total erosion March 1960[a]			Increase in erosion Dec 57-Mar 60[a]		
	Average, feet	Minimum, feet	Maximum, feet	Average, inches	Minimum, inches	Maximum, inches
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) Row of slabs containing baffle piers						
1-A	0.41	0.19	1.71	1-3/4	0	4
2-A	0.82	0.26	2.46	1-1/4	0	8
3-A	0.91	0.25	2.43	2-1/4	0	5-1/2
4-A	0.30	0.12	0.53	b	b	b
5-A	0.25	0.11	0.43	1-1/2	1/2	2
6-A	0.33	0.08	0.91	1/2	0	2-1/2
7-A	0.97	0.40	5.05	4	0	10
8-A	0.44	0.25	1.39	1-1/2	0	6
9-A	0.71	0.14	3.14	1-1/2	0	3-1/2
10-A	1.21	0.28	4.85	2-1/4	0	37
11-A	0.64	0.23	1.98	1-3/4	0	4-1/2
12-A	0.97	0.20	3.81	1	0	3
13-A	0.69	0.07	2.03	3/4	0	3-1/2
14-A	0.42	0.11	1.50	1	0	4-1/2
Averages	0.65			1-5/8		
(b) First row of slabs upstream from baffles						
1-B	0.32	0.02	1.26	1-1/2	0	4
2-B	0.52	0.24	1.45	1-1/4	0	3
3-B	0.48	0.15	1.31	2	0	3
4-B	0.37	0.25	0.45	b	b	b
5-B	0.20	0.09	0.35	1/4	0	1-1/2
6-B	0.37	0.19	0.79	3/4	0	3
7-B	0.33	0.21	0.54	1-1/4	0	4
8-B	0.39	0.28	0.54	1-3/4	1/2	3
9-B	0.37	0.19	0.74	1-1/2	0	3
10-B	0.64	0.34	1.09	2-1/2	0	6
11-B	0.52	0.31	0.93	2-1/4	0	4-1/2
12-B	0.56	0.24	1.34	1-1/2	0	3-1/2
13-B	0.28	0.18	0.49	1-3/4	0	3-1/2
14-B	0.36	0.13	0.66	2-1/2	0	4
Averages	0.50			1-5/8		
(c) Second row of slabs upstream from baffles						
1-C	0.20	0.06	0.56	1-1/2	1/2	3
2-C	0.23	0.14	0.43	1	0	2
3-C	0.22	0.14	0.49	b	b	b
4-C	0.41	0.25	1.09	2	0	3-1/2
5-C	0.36	0.22	0.92	1/2	0	2-1/2
6-C	0.27	0.15	0.50	1/2	0	2-1/2
7-C	0.34	0.09	0.64	1/2	0	2
8-C	0.19	0.07	0.29	1/4	0	1-1/2
9-C	0.33	0.19	0.64	1-3/4	0	4
10-C	0.28	0.11	0.66	2-1/4	0	4
11-C	0.31	0.09	0.84	1-3/4	0	3
12-C	0.25	0.09	0.54	1-3/4	1/2	3
13-C	0.26	0.11	0.46	1-3/4	1	2-1/2
14-C	0.28	0.07	0.51	2-1/2	1/2	3-1/2
Averages	0.28			1-3/8		

[a] Does not include erosion measurements taken in construction joints which are not considered representative of slab.

[b] Visual inspection only, 1957 (except several isolated measurements).

[c] Visual inspection made in 1957; no measurements were taken.

occurred principally on the lower front face and sides with generally accompanying damage to adjacent slab areas. However, of the solid shaded baffles, numbers 4, 26, 27, and 28 suffered damage principally to the tail of the baffle, and number 17 had a large piece knocked out of an upper corner.

SURVEY OF MARCH, 1960

To establish the rate of progression of erosion damage and to determine whether erosion might be attributable to operating conditions, a second underwater survey was undertaken in March, 1960. The results of this survey tabulated in terms of total average, minimum and maximum erosion in feet of each slab in 1960 and the corresponding increase in erosion in inches since 1957 are shown in Fig. 6 and in Table 2. In the two flood seasons between surveys, peack flows of 377,000 in 1958, and 395,000 in 1959, were experienced, both of which are less than 4-yr frequency occurrences. It was not possible to make measurements in the "D" row of slabs in 1960 because of the hazard of ice falling off the tainter gate assemblies. During the survey, the divers were able to distinguish many light colored fresh pockets throughout the basin in which aggregate had been recently eroded from the surface of the concrete. There was also extensive occurrence of fresh impact marks on baffles and end sill, probably caused by small rounded balls of aggregate found in the basin.

The average increase in erosion between the 1957 and 1960 survey in each of rows "A," "B" and "C" was found to be approximately 1-1/2 in., although the total erosion in both "A" and "B" rows of slabs appreciably exceeded that in upstream row "C." Correspondingly, the 2-yr increase in damage relative to the initial damage measured in 1957 is much greater in the upstream row of slabs. Additional surveys in the future will be needed to determine whether this trend will continue.

In the 1960 survey, detailed measurements were made of only fourteen baffle piers including ones representative of light, moderate and heavy damage in 1957. In general, baffle piers with surfaces that were undamaged or in sound condition in 1957 were also found to be in sound condition in 1960. Increases in baffle pier erosion was limited to surfaces that were found to have been markedly eroded in 1957.

Changes in end sill erosion followed the same condition pattern as the baffle piers with added erosion being generally limited to some areas that had previously been damaged. The overall average increase in erosion of damaged end sill areas was only 3/8 in.

CAUSE OF DAMAGE

The damage observed in the last 2 yr of spillway service appears minor as compared to the pattern of major damage found in the 1957 survey. It appears that most of the damage to the stilling basin has originated from conditions during project construction. The pattern of localized, deep and extensive erosion between the end sill and certain baffles and the damage to front faces and sides of these baffles indicates that much of the damage must have resulted from scouring of foreign materials in the basin, particularly, because there has not been any development of new areas of severe damage

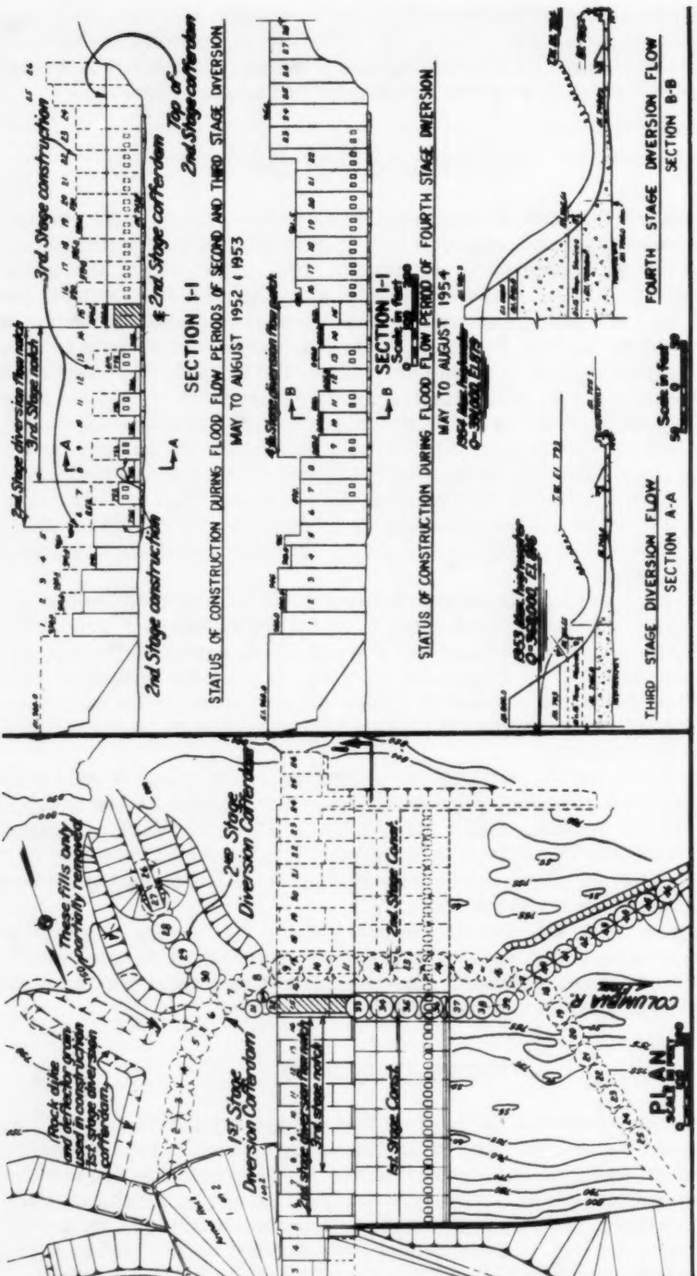


FIG. 7.—DIVERSION DETAILS

since 1957. An analysis of conditions during project construction has verified that on several occasions, rock was introduced into the stilling basin during river diversion, and this situation combined with diversion flow conditions created a potential for major damage.

Fig. 7 is a composite drawing showing the several stages of cofferdam construction for river diversion. In the first stage of diversion a cofferdam consisting of a series of steel sheet pile circular cells, numbered 1 through 25, were installed to divert the river to the left side of the river channel. During the construction of these cells it became necessary to dump a substantial groin of 1-ton to 3-ton rock immediately upstream of cells 3 through 5 in order to divert high velocity river currents away from the area of construction of downstream cells. This rock groin, shown on Fig. 8, was only partially removed. Five low monoliths approximately at streambed level were constructed in the first stage cofferdam together with four pairs of 8-ft-wide by 16 ft-high temporary sluices with invert elevations 14 ft above riverbed for use in passage of second stage diversion flows.

First stage construction was completed in January, 1952; the cells were removed and second stage diversion, consisting of cells numbered 26 through 46, were installed and operative by April, 1952 so that the 1952 high-water was entirely diverted through the low monoliths and temporary sluices of the first stage construction on the right side of the channel. Fig. 9 shows flow conditions in March, 1952, with the second stage cofferdam nearly completed just prior to high water. Note the white water over the remains of the rock groin upstream of the first stage construction and also the outside rock groin being placed at the upper end of the second stage diversion cofferdam. In addition to rock wash from these sources, a near failure of cell 29 in the second stage cofferdam during the 1952 high-water required emergency dumping of rock around the outside of the cell. The high velocity flow around the outside of the cofferdam undoubtedly washed a considerable amount of rock into the first stage stilling basin. There was also rock wash into the basin from placement of armor toe rock in the wraparound abutments, particularly on the right bank side. With this set of circumstances, the pattern of concentrated damage downstream from the low block monoliths 6, 8, 10, 12, and 14 becomes understandable.

In the third stage of diversion in 1953, Fig. 10, the cellular cofferdams had been removed and flow was diverted through the twenty-four temporary sluices and the low monoliths that were still at streambed level. During flood periods a very heavy concentration of flows resulted through the low monolith diversion notch that, in turn, generated a strong eddy current in the basin. This same pattern of flow was repeated in 1954, except that with the raising of the monoliths the diversion notch was now 100 ft above the stilling basin floor and the corresponding maximum head was 136 ft from pool to stilling basin floor. Fig. 11 shows flow through the fourth stage diversion notch and the location of eddy currents.

The significance of eddy current action on erosion of the stilling basin is illustrated by Fig. 13, that shows exposed reinforcing bars and erosion on the streambed face of the end sill with the end sill concrete eroded to expose reinforcing bars bent upstream counter to the direction of flow. It is apparent that the eddy current was of sufficient strength to pick up rock from the streambed below the dam and carry it into the basin during the third or fourth stage diversion.

FIG. 8.—FIRST STAGE COFFERDAM CONSTRUCTION SHOWING ROCK GROIN USED AS CURRENT DEFLECTOR

FIG. 9.—SECOND STAGE COFFERDAM PRIOR TO OCCURRENCE OF HIGH WATER AND SHOWING WHITE WATER OVER ROCK GROIN AND BERM CONSTRUCTION

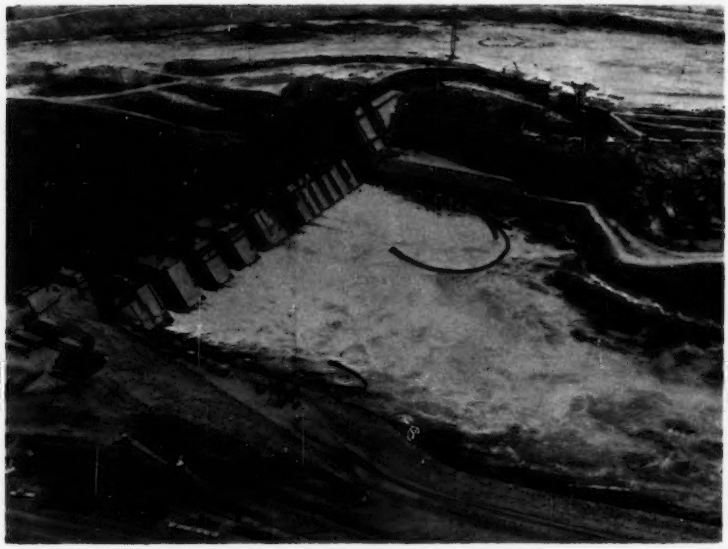


FIG. 10.—THIRD STAGE CONSTRUCTION DURING HIGH-WATER SHOWING EDDY CURRENTS

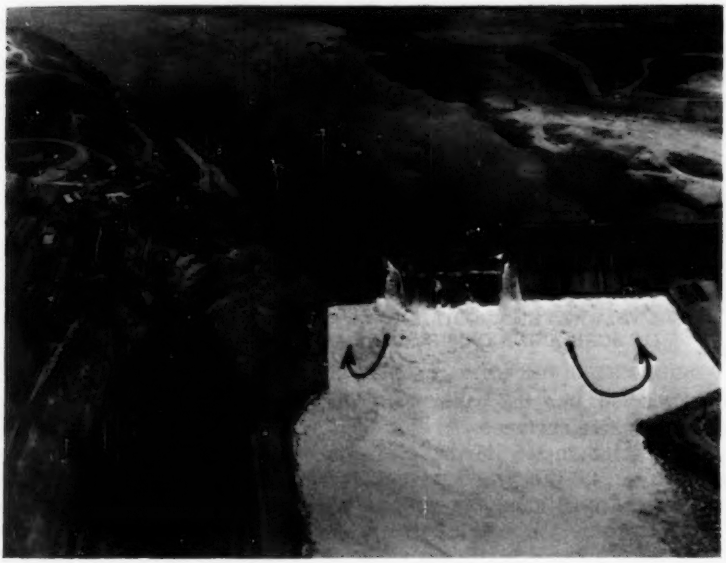


FIG. 11.—FOURTH STAGE CONSTRUCTION DURING HIGH WATER SHOWING EDDY CURRENTS

The initial underwater inspection made in 1957, two years after the spillway was placed in full service, showed no rock in the basin other than a small amount of dislodged aggregate. This confirmed model tests that showed the completed basin in normal operation to be self-cleaning for discharges in excess of 300,000 cfs. However, flow conditions throughout the diversion period were so different from normal operation that it is probable

FIG. 12.—UNDERWATER PHOTOGRAPH SHOWING ROUGHENED CONDITION OF SLAB AND EXPOSURE OF LARGE AGGREGATE

that rock washed into the basin was retained as a tumbling agent to scour and abrade concrete surfaces.

Another damage factor during diversion was that the very heavy concentration of flows in the fourth stage diversion notch approached maximum spillway design conditions, but with an appreciably greater tailwater deficiency than that for which the basin was designed, as indicated in Table 3.

This lack of adequate tailwater depth created a temporary condition for potential cavitation damage during diversion.

CONCRETE QUALITY

The possibility of stilling basin damage because of the quality of the concrete has been considered. Concrete for the stilling basin was designed

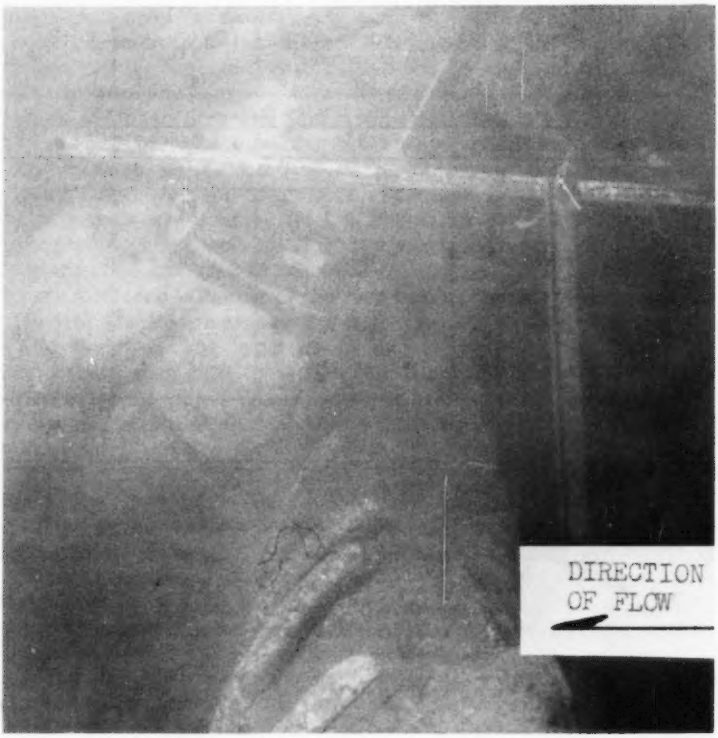


FIG. 13.—UNDERWATER PHOTOGRAPH SHOWING REINFORCING BARS IN END SILL BENT UPSTREAM.

for a compressive strength of 3,000 psi. However tabulation of concrete properties from construction records (Table 4) indicates, this strength requirement was substantially exceeded in many areas. Slab and unformed surfaces of the top of baffles, end sill and ogee bucket section received a steel trowel finish. Smooth-surfaced tongue and groove lumber was used for formed sections of the baffles and end sill. Concrete was water-cured for 14 days. A large proportion of concrete in the north half of the stilling

basin was placed during freezing weather with protection from freezing provided by tarpaulins salamanders. No special quality of concrete surface was specified. Three possible origins of damage to concrete other than rock abrasion are believed possible.

Erosion of Concrete by Water-Jet Action.—The exposed aggregate condition of the stilling basin floor and concave wearing at the base of and on the

TABLE 3.—COMPARISON OF STILLING BASIN DESIGN AND FOURTH STAGE DIVERSION ENERGY DISSIPATION CHARACTERISTICS

Total flow (1)	Q per foot, cfs (2)	Entrance velocity, feet per second (3)	D_2'/D_2 (4)	$D_2' - D_2$, feet (5)
		Spillway Design		
400,000	440.	113.	0.95[a]	1.0
		Fourth Stage Diversion		
391,000 (peak)	1140.	89.	0.71[a]	23.0
350,000 (exceeded 44 days)	1020.	88.	0.70[a]	21.0
300,000 (exceeded 66 days)	875.	87.	0.73[a]	18.0

[a] D_2 is theoretical depth of tailwater for jump formation.
D_2' is actual depth of tailwater for jump formation.

TABLE 4.—PROPERTIES OF CONCRETE USED

Location (1)	Maximum Size Aggregate, in inches (2)	Cement Content in sachs per cu yd (3)	Water content ratio by weight (4)	Entrained Air,[a] percent (5)	Slump at plant, inches (6)	Compressive Strength,[b] in psi: 28 days (7)	90 days (8)
Apron Slab	6	4	0.44	6	2	4,500	5,600
Baffles and end sill	3	4	0.49	4	2	3,600	5,000

[a] minus 1 1/2 in.
[b] Approximate overall average: cylinder tests

front face of many baffles could possibly be attributed to erosion caused by high velocity water-jet action. High velocity flows through the stilling basin during diversion have ranged from 30 fps to over 100 fps. Many of the changes in the direction of flows were induced by impact, either from flow over monoliths, through temporary sluices striking the ogee or stilling basin floor, or from diffusion of flows by baffles.

The first erosion due to water-jet action was reported during the initial diversion of high water in 1952 when water flowed up to 15 ft deep over monoliths 7, 9, 11, and 13. Inspection immediately after the flood-flow period revealed light scour and an exposed aggregate condition of concrete on these monoliths below Ele. 775. Extensive tests conducted by the United States Bureau of Reclamation, Dept. of Interior indicates that concrete will stand the flow of clear water at high velocities without appreciable damage, provided that the flow is smooth and the direction and velocity of the water is not abruptly changed. However, water-jet tests at velocities from 100 fps to 175 fps, showed pitting and surface roughening when the jet angle exceeded 5°. Damage from scour at moderate to high velocities is apparently a random occurrence depending on the density and quality of the surface attained in construction. It is expected that resistance to high velocity scour varied throughout the basin as no special quality of finish was specified.

Exposure of Under-Strength Concrete to High Velocity Flows.—Some concrete in the north half of the stilling basin was exposed to high velocity river flows ranging up to 40 fps within 60 days after placement. Concrete strength at this time may have been below normal because it was placed and cured in freezing and subzero weather during November 1951 and December 1951. Some concrete was exposed to 45 F temperatures and colder, high velocity flows less than one month after placement.

Concrete Surfaces.—Paving of the slab using concrete with 6-in maximum size aggregate resulted in exposure of large size aggregate with minor erosion of the slab surface. The exposure of this aggregate created an immediate cavitation hazard that with cavitation damage could spread progressively to undamaged adjacent areas. The model tests showed incipient cavitation damage for minor deviations from the theoretical curvature on the sides of the baffles even when using a smooth curve. In the prototype, side curves were formed with narrow tongue and groove lumber, resulting in a marked deviation from the type of surface tested in the model. It is apparent that any irregularities in the surface resulting from forming, finishing, or patching would greatly increase the possibility of cavitation. In construction, the baffles were formed only on the sides and front with the top and back slopes placed and finished without forms. The use of this open-type forming limited the amount of vibration and compaction of the concrete because of sloughing of the back slope, thus creating conditions that caused rock pockets on formed areas. Although all such areas were patched, these patches are areas of weakness and could have created irregularities or roughness to start cavitation.

REMEDIAL MEASURES TO PREVENT OR MINIMIZE DAMAGE IN FUTURE CONSTRUCTION

The primary cause of stilling basin damage at Chief Joseph Dam is believed to be scour caused by large size rock particles introduced into the basin by diversion flow conditions during construction. It is also possible that damage could have been reduced or minimized by some higher quality control of concrete surfaces.

A contributing factor to the occurrence of this damage was undoubtedly a lack of awareness of the damage potential during construction at both design and field levels. Full consideration of the problem during the model testing stage might have led to changes in design. For example, the tendency of the

stepped-end sill to retain rock could have been reduced by sloping the face of the sill. It might have been possible to raise the elevation of the low monoliths 10 ft to 15 ft above streambed to prevent rock from entering the basin during diversion, but this would have necessitated higher cofferdams and the single cell-type of cofferdam used was already at maximum height for the strength of steel sheet pile that was available. Thus, a double cell-type cofferdam probably would have been required.

A full consideration of the problem might have indicated a different plan of diversion during construction; however, the costs might well have exceeded costs of repairing damage to the basin. Greater restrictions placed on the operations of the contractor to minimize operations that introduced rock into the stilling basin would also have increased the costs. A higher quality concrete surface would have been desirable and steps were taken several years ago by the Corps of Engineers to specify superior quality concrete of higher strength and better finish and employing small size aggregate for use in structures with high velocity flows.

The steps taken to control damage during construction must necessarily be tempered by balancing ultimate costs of repair against the added costs of damage-preventative construction measures.

CONCLUSIONS

Damage to the Chief Joseph Dam stilling basin reported herein was caused primarily by rock entering the basin during the diversion phase of construction. A greater awareness of this type of damage potential by both designers and field forces is necessary to avoid future recurrences.

ACKNOWLEDGMENT

The data used herein are taken from records of the Seattle District, Corps of Engineers. Work on stilling basin studies has been under the direction of R. P. Young, Colonel, U. S. Army and District Engineer; and Noble A. Bosley, Chief, Engineering Division. All of the detailed under water surveys and development of procedures for these surveys have been accomplished under the technical supervision of W. W. Whipple of this office.

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FORECASTING RIVER RUNOFF BY COASTAL FLOW INDEX

By David M. Rockwood,[1] M. ASCE, and Carlton E. Jencks[2]

SYNOPSIS

An index method for forecasting April through September runoff for the Columbia River at The Dalles, Oreg., is presented, based on the relationship between winter runoff in coastal drainage areas and spring snowmelt runoff of the Columbia River. Reliability of the forecasts is comparable with those based on precipitation and snow-course data.

INTRODUCTION

The Columbia River is one of the principal snowmelt runoff streams in the United States. Inasmuch as the flow of the Columbia River is primarily from snowmelt, an evaluation of existing snowpack water equivalent makes it possible to forecast the total runoff potential several months in advance. Such forecasts enhance the use of multiple-purpose reservoir storage for obtaining optimum benefit of water resources for all water uses. The evaluation of the total runoff potential of the Columbia River is necessary for coordinated system operation of reservoir storage for flood control and other water use requirements, as well as for flood potential evaluation prior to completion of an adequate system of reservoirs for flood control. The Corps of Engineers, United States Army, is responsible for flood flow regulation in the Columbia River Basin. The prime basin forecast of the total runoff is for the gaging station located at The Dalles, Oreg. (D.A. = 237,000 sq miles), at which point

Note.—Discussion open until August 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 2, March, 1961.

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the runoff from essentially the entire Columbia River drainage east of the Cascade Range is measured.

Methods of forecasting seasonal runoff volume from snowmelt have been based primarily on use of precipitation or snow accumulation measurements. The most commonly used procedures employ the concept of indexes, whereby point measurements of hydrologic variables are correlated with runoff by various statistical techniques. Coefficients so derived from historical data provide the basis for forecasting future runoff, by applying them to observed index values. Forecasting methods of this type depend upon an adequate period of record for statistical treatment, and these index procedures may involve several variables affecting runoff.

One of the objectives of the Corps of Engineers Weather Bureau, United States Dept. of Commerce (USWB), Cooperative Snow Investigations, was the development of practicable and reliable methods of forecasting seasonal streamflow resulting from snowmelt. The emphasis in the analytical work that was carried on by the Corps of Engineers' Snow Investigations Unit in pursuit of this objective was in the rational evaluation of the water balance on basin areas involving snow, and the determination of the storage of water in the snowpack in relation to the other factors involved in the hydrologic cycle. The results of these investigations are available.[3,4]

During the course of the snow investigations, it was realized that index procedures are useful in deriving practical forecasting equations. General knowledge of the hydrologic cycle and causative meteorologic factors involved in the deposition of precipitation, as applied to a specified basin, are essential for the intelligent selection of suitable parameters for forecasting seasonal runoff by index methods.

Due to the fact that the Columbia River Basin is located in the middle latitudes, in the zone of the prevailing westerly air flow, the sources of moisture supply are the maritime air masses of Pacific origin, particularly during the winter season. Precipitation occurs with the eastward movement of large-scale frontal systems from the Pacific. These systems, which are usually occluded by the time they reach the coast, produce varying amounts of frontal precipitation. Additional precipitation results from orographic lifting of the moist air masses as they pass over the Coast, Cascade, and Rocky Mountain topographical barriers. Inspection of upper-air charts[5] shows that, during times of significant winter precipitation over the Columbia River Basin, the average air flow at the 500 and 700-mb level is generally from the west, but may vary from southwest to northwest. Further inspection reveals that the basin is generally well centered in the westerly stream of air which, on the average, covers a width of about 15° to 20° of latitude.

The initial orographic uplift of the air stream over the Coast and Cascade Ranges results in deposition of a portion of the total precipitable water of the air mass before it enters the Columbia River Basin, east of the Cascades. The lower elevations and predominant maritime air-mass characteristics of the drainage areas in the Coast and Cascade Ranges result in higher tempera-

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3 "Summary Report of the Snow Investigations, Snow Hydrology," U. S. Corps of Engrs., North Pacific Div., Portland, Ore., June 30, 1956.

4 "Application of Snow Hydrology to the Columbia Basin," by Oliver Johnson and Peter B. Boyer, Proceedings, ASCE, Vol. 85, No. HY 1, January, 1959.

5 U. S. Weather Bur., daily series synoptic weather maps, northern hemisphere sea level and 500 millibar charts, U. S. Dept. of Commerce, Washington, D. C.

tures which in turn cause a large part of the precipitation to fall as rain, while in the higher and more continental areas of the Columbia Basin nearly all of the precipitation falls as snow.

The winter runoff from the Coastal streams is considered to be a reliable index of the total winter precipitation falling on the Columbia River Basin, because it samples the total incoming moisture supply in the air masses during times of strong zonal flow. Such an index applied to the Columbia Basin as a whole would be a measure of the total moisture supply to the basin and therefore may be used as a forecasting parameter. However, it would not evaluate the distribution of precipitation on the various tributary streams or sub-basins by the application presented herein for the Columbia River Basin.

With this concept in mind, a project was carried on in 1954 as part of the work of the Snow Investigations Unit, whereby forecasting relationships were derived for the Columbia River at The Dalles, using graphical correlation techniques. The period of record used in those studies was the 22-yr period of 1929 through 1950. A report on the study has been issued.[6] Subsequent to that time, nine additional years of record have become available for study (1951 through 1959). Furthermore, the North Pacific Division office acquired use of an electronic digital computer in 1956, which could be utilized for multiple-regression analysis. Accordingly, the analysis presented herein has taken advantage of these conditions and is independent of that data previously presented,[6] although much of the basic data is utilized.

OBJECT AND SCOPE

A set of correlations of runoff with a coastal winter-flow index and other significant runoff parameters, by which runoff of the Columbia River at The Dalles for the period April through September may be determined will be presented. Both winter and springtime effects on runoff are evaluated, and results derived by arithmetic statistical methods are presented in graphical form, for the evaluation of effectiveness of the parameters in estimating runoff, and the range of values normally experienced. The accuracy of forecasts is briefly compared with other methods of forecasting in common use in the Columbia Basin.

Considerable use is made of statistical techniques in deriving the relationships presented herein. It is assumed that the reader is acquainted with basic statistical terminology and analytical procedures, and for the sake of brevity, no general summary of statistical methods is included. For those who desire information of this nature, textbooks on statistical analysis are available.[7]

DATA

Columbia River Near The Dalles, Oreg.—The United States Geological Survey, Dept. of the Interior (USGS) has published continuous records of stream-

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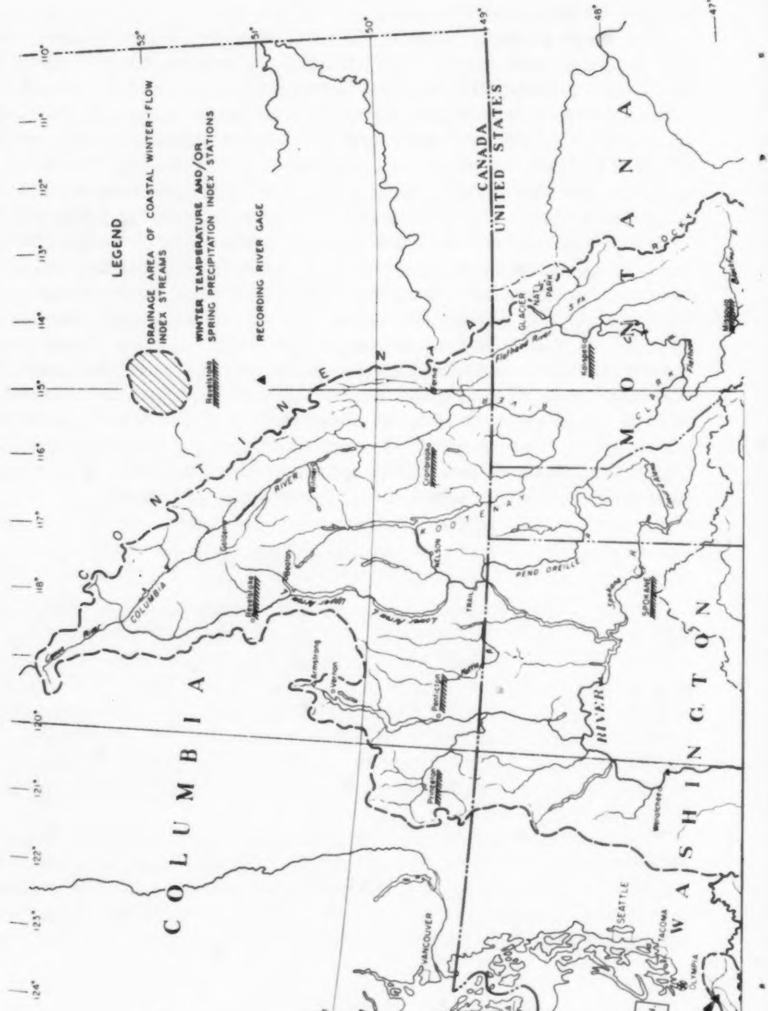
6 "A Coastal Flow Index Method of Forecasting Seasonal Runoff for Columbia River near The Dalles, Oregon," by David M. Rockwood, Snow Investigations Research Note Note No. 23, U.S. Army Engr. Div., North Pacific Div., Portland, Oreg., September 30, 1954.

7 "Statistical Methods," by Herbert Arkin and Raymond R. Colton, (College Outline Series), Fourth Ed., Barnes and Noble, Inc., New York, N. Y., 1955.


LEGEND
DRAINAGE AREA OF COASTAL WINTER-FLOW INDEX STREAMS
WINTER TEMPERATURE AND/OR SPRING PRECIPITATION INDEX STATIONS
RECORDING RIVER GAGE
BRITISH COLUMBIA
CANADA
UNITED STATES
MONTANA
WASHINGTON
CONTINENTAL
COLUMBIA
KOOTENAY
PEND OREILLE
NELSON
TRAIL
SPOKANE
GLACIER NAT'L PARK
Flathead River
Kalispell
Cranbrook
Revelstoke
Penticton
Princeton
VANCOUVER
VICTORIA
SEATTLE
TACOMA
OLYMPIA
STRAIT OF JUAN DE FUCA
CHEHALIS R. NR. GRAND MOUND, WN. D.A. = 895 SQ. MI.

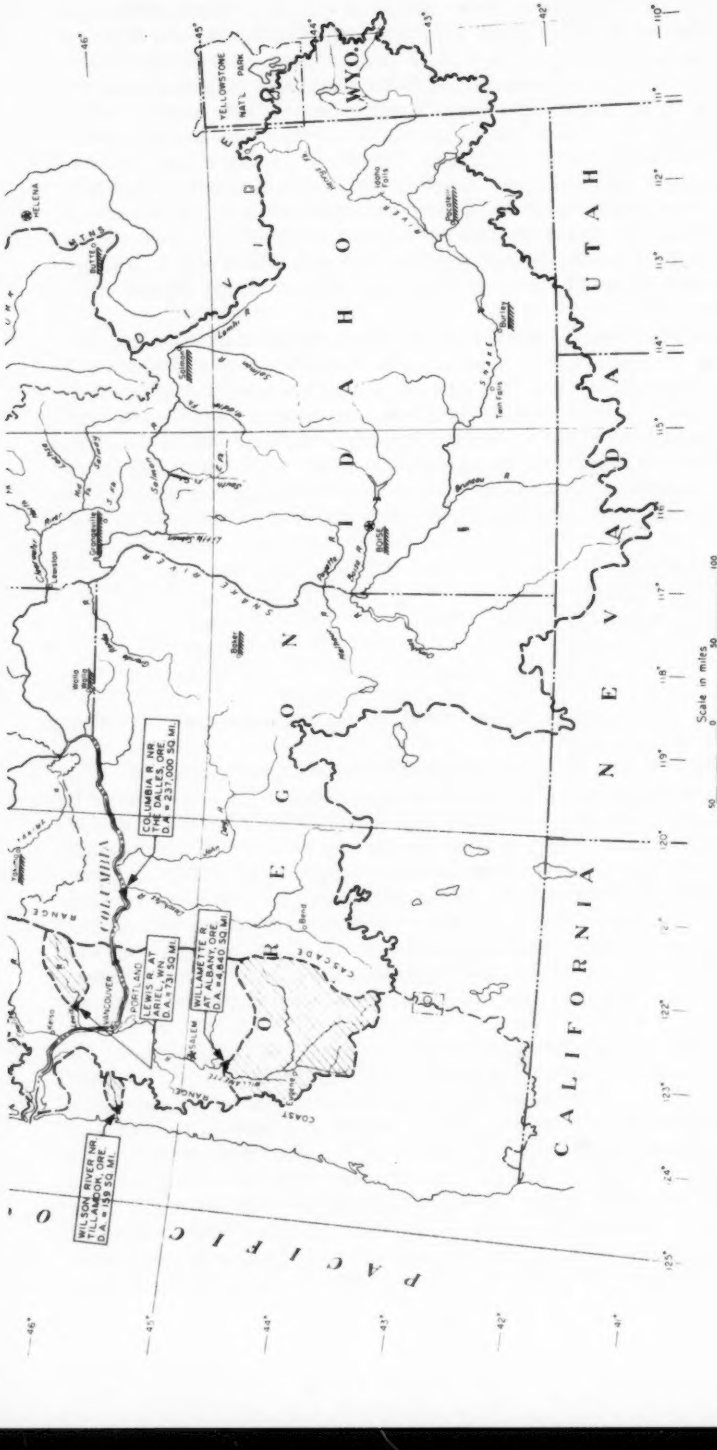


FIG. 1.—LOCATION MAP

flow for the Columbia River near The Dalles, Oreg., since June, 1878. The record for that station is the longest in the Pacific Northwest and is rated "excellent" by the Geological Survey. It is the furthest downstream of any rated gaging station on the main stem of the Columbia River, and the drainage area is 237,000 sq miles. Annual and April-through-September runoff volumes were determined from this record. Appropriate corrections for major reservoir storages, including Kootenay, Hungry Horse, Flathead, Pend Oreille, Franklin D. Roosevelt, Coeur d'Alene, Chelan, and Brownlee, were applied to observed flows. There are many irrigation diversions and small reservoir storages which affect the flows at The Dalles, but their net effect on annual and seasonal runoff is small in comparison with the total volume. This is particularly true with regard to year-to-year differences. Their effects were therefore ignored.

Coastal Winter-Flow Index.—It has been pointed out that the use of a coastal winter-flow index is predicated on evaluation of runoff from areas which are at low elevations relative to the Columbia Basin as a whole, and also which are well located with respect to the air stream during average winter air-flow conditions. In addition, records of sufficient length for analysis of data are required. The 31-yr period 1929 through 1959 was chosen for study, principally on the basis of continuity of streamflow records for coastal streams. The combination of streams selected for the coastal winter-flow index is as follows:

Stream-Gaging Station	Drainage Area, Sq Miles
Willamette River at Albany, Oreg.	4,840
Wilson River near Tillamook, Oreg.	159
Lewis River at Ariel, Wash.	731
Chehalis River near Grand Mound, Wash.	895

Fig. 1 is a map of the Columbia River Basin and the coastal region, showing the location of the index streams.

The Willamette River at Albany was selected because of its long period of record and because it samples the flow from the Cascade Mountains as well as the coast range. Due to its moderately high average winter temperatures, nearly 70% of the annual runoff occurs during the period October through March. Records for the station are termed "good" by the USGS. For the period through 1950, no correction was incorporated for reservoir storage because of the relatively small amount of reservoir capacity above the station (about 140,000 ac-ft of storage out of 10,000,000 ac-ft annual runoff). For the period subsequent to 1950, appropriate corrections for storage in Fern Ridge, Cottage Grove, Dorena, and Lookout Point Reservoirs were applied to observed streamflows.

Wilson River near Tillamook samples the air flow north of the Willamette River at Albany, and it represents runoff from the west side of the coast range. Only 6% of its area is above 3,000 ft and about 80% of its annual runoff occurs from October through March. The records are termed "good," and there are no upstream diversions or regulation. Lewis River at Ariel was used for sampling a west slope Cascade drainage area in Washington. Its average elevation is about 2,800 ft, somewhat higher than the other index drainages, and about 62% of its annual runoff occurs from October through March. The records are termed "good," and appropriate corrections were made for changes in reservoir storage. A lower elevation area in Washington

is represented by Chehalis River near Grand Mound. Here only 1% of its area lies above 3,000 ft, and 81% of the annual runoff occurs from October through March.

Table 1 shows average monthly runoff for the period October through March, during the period 1929 through 1959, for each of the four coastal winter-flow index stations. Table 2 shows the total October-through-March flow for each station, expressed in percentage of the 22-yr average for the period 1929 through 1950. The four station average is indicated in column 6, Table 2. A minor recession correction, for the purpose of correcting the observed runoff for water temporarily in storage in the coastal flow index streams at the end of March, is shown in column 7. This correction was determined by recession analysis for Willamette River at Albany, in a manner similar to that shown elsewhere.[8] The coastal winter-flow index (CFI) values shown in column 8, Table 2, are those used in the multiple linear regression analysis to be described.

Columbia Winter Runoff Correction Index.—Nearly all of the winter precipitation from significant runoff producing areas in the interior of the Columbia River Basin falls as snow. Streamflow during the winter is largely base flow recession, augmented by occasional low-level rain and snowmelt runoff. Only 28% of the annual runoff occurs in the period October through March. Nevertheless, significant variation in winter runoff does occur as the result of unusual weather conditions. Inasmuch as the basin moisture input indexes were correlated directly with April through September runoff for the Columbia River at The Dalles, it was necessary to account for the variability of winter runoff. The October through March runoff for the Columbia River at The Dalles, corrected for storage in Kootenay, Hungry Horse, Flathead, Pend Oreille, Coeur d'Alene, Franklin D. Roosevelt, Chelan, and Brownlee Reservoirs is tabulated in Table 3. The winter runoff correction index is the product of the October through March runoff and the costal winter-flow index (CFI); the values are tabulated in Table 3.

Winter Temperature Correction Index.--In order to account for differences in evapotranspiration losses, a winter temperature index is included in the analysis. The index used in this procedure is a 5-station mean for the months of October, November, February, and March. Climatological records for Revelstoke, British Columbia; Missoula, Mont.; Spokane, Wash.; Yakima, Wash.; and Boise, Idaho, were used for this purpose, and values of mean monthly temperature are tabulated in Table 4. The temperature correction index is the product of the 5-station, 4-month mean temperature, times the coastal winter-flow index. The locations of the temperature index stations are shown on Fig. 1.

April Coastal Flow Correction Index.—April 1 of each year is a convenient point of separation between winter and spring effects of runoff in the Columbia River Basin. Springtime effects are considered in this study to be confined to the months of April, May, and June. Precipitation during April is normally associated with the zonal atmospheric flow pattern characteristic of the winter season. Accordingly, the index of moisture supply during April was the coastal-flow runoff. In this case it is only for the Chehalis River near Grand Mound, Wash. Only the one station was used because the index streams of the Willamette and Lewis Rivers may have significant amounts of snowmelt runoff

[8] "Hydrology Handbook," ASCE, Manuals of Engrg. Practice, No. 28, New York, 1949, p. 72.

TABLE 1.—MONTHLY RUNOFF IN 1,000 ACRE-FT[a]

Water Year (1)	October through December (2)	January (3)	February (4)	March (5)
(a) WILLAMETTE RIVER AT ALBANY, OREG. D. A. = 4,840 Sq Miles[a,b]				
1959	2,548	2,274	1,548	1,011
1958	2,790	2,125	2,631	1,157
1957	2,877	788	1,582	2,654
1956	6,692	3,413	1,394	1,827
1955	1,452	1,337	815	1,334
1954	4,376	2,428	1,952	986
1953	942	3,241	2,378	1,328
1952	4,136	1,456	1,995	1,335
1951	5,848	2,892	2,105	1,580
1950	1,550	2,236	2,219	2,082
1949	3,540	714	2,134	1,444
1948	3,961	2,518	1,635	1,411
1947	3,793	1,026	1,292	1,168
1946	3,553	2,522	1,236	1,784
1945	750	1,015	1,787	1,494
1944	1,695	700	898	695
1943	4,854	3,100	1,786	880
1942	3,456	1,247	1,199	670
1941	1,768	1,181	550	421
1940	945	750	1,870	1,533
1939	1,640	969	1,468	1,541
1938	3,633	1,869	1,818	2,301
1937	583	407	1,545	1,538
1936	970	2,852	1,178	1,121
1935	3,437	1,670	1,120	1,241
1934	2,126	1,816	592	726
1933	2,081	1,960	1,380	1,520
1932	2,093	1,750	1,020	2,500
1931	889	627	472	1,090
1930	1,510	569	1,930	707
1929	1,081	1,060	739	1,070
Average, 1929-1950	2,269	1,480	1,358	1,315
Accumulated Average for Period Ending	2,269	3,749	5,107	6,422
(b) WILSON RIVER NEAR TILLAMOOK, OREG. D. A. = 159 Sq Miles[a]				
1959	330	211	88	89
1958	297	186	160	56
1957	252	51	133	133
1956	600	214	65	224
1955	270	109	102	119
1954	379	210	197	75
1953	118	355	129	88
1952	394	105	141	93
1951	425	215	163	103
1950	320	188	195	202
1949	382	43	237	106
1948	374	150	149	99
1947	429	119	124	74
1946	387	185	152	122

TABLE 1.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)
(b) WILSON RIVER NEAR TILLAMOOK, OREG. D. A. = 159 Sq Miles[a]				
1945	131	138	141	154
1944	179	95	79	66
1943	404	101	162	86
1942	326	62	99	58
1941	175	127	42	37
1940	230	94	190	114
1939	207	102	148	95
1938	488	132	88	140
1937	142	58	148	129
1936	101	227	90	88
1935	555	189	86	125
1934	633	229	40	108
1933	399	180	112	178
1932	321	169	127	204
1931[c]	92	102	50	135
1930[c]	132	66	225	94
1929[c]	232	111	56	84
Average, 1929-1950	302	130	124	114
Accumulated Average for Period Ending	302	432	556	670
(c) LEWIS RIVER AT ARIEL, WASH. D. A. = 731 Sq. Miles[a,b]				
1959	1,320	718	263	334
1958	888	558	598	278
1957	1,107	191	314	551
1956	1,926	596	225	496
1955	840	284	314	194
1954	1,263	469	538	350
1953	301	1,153	521	296
1952	1,263	193	441	271
1951	1,658	505	576	252
1950	1,035	448	520	687
1949	898	117	370	419
1948	1,297	488	356	284
1947	1,569	363	442	330
1946	1,035	547	336	410
1945	459	475	424	337
1944	572	251	251	212
1943	1,181	285	366	369
1942	1,129	196	284	216
1941	676	346	188	174
1940	692	261	599	519
1939	688	442	308	330
1938	1,461	512	222	390
1937	443	102	161	401
1936	428	684	225	335
1935	1,472	411	336	294
1934	2,379	1,025	268	446
1933	1,194	502	184	437
1932	781	399	338	756

TABLE 1.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)
(c) LEWIS RIVER AT ARIEL, WASH. D. A. = 731 Sq Miles[a,b]				
1931	306	324	265	442
1930	377	170	628	283
1929	667	212	108	306
Average, 1929-1950	943	389	326	381
Accumulated Average for Period Ending	943	1,332	1,658	2,039
(d) CHEHALIS RIVER NEAR GRAND MOUND, WASH. D. A. = 895 Sq Miles[a]				
1959	780	558	257	214
1958	518	397	383	162
1957	616	139	335	398
1956	1,502	602	220	557
1955	554	313	243	300
1954	848	597	617	194
1953	174	787	350	168
1952	781	271	349	189
1951	1,077	590	516	310
1950	735	500	593	522
1949	1,006	131	614	243
1948	704	422	368	263
1947	878	314	361	158
1946	572	484	429	282
1945	242	296	346	378
1944	290	255	186	140
1943	711	252	385	165
1942	763	173	234	137
1941	405	284	105	90
1940	405	234	522	333
1939	368	394	454	225
1938	1,066	358	233	325
1937	259	133	436	261
1936	186	634	296	340
1935	1,158	593	219	315
1934	1,462	581	112	212
1933	1,045	508	243	426
1932	680	371	383	451
1931	168	347	214	290
1930	214	127	379	191
1929	257	172	128	242
Average, 1929-1950	566	344	329	272
Accumulated Average for Period Ending	566	910	1,239	1,511

[a] Data from U. S. Geol. Survey Water Supply Papers, 1929-1956 unpublished records by U. S. Geol. Survey, 1957-1959

[b] Stream flow corrected for reservoir storage

[c] Estimated by correlation with Silete River at Silete, Oreg.

during the month of April, while the Chehalis River is virtually unaffected by snowmelt runoff during April. Table 5 lists the April runoff for Chehalis River near Grand Mound, together with the recession correction applied at the beginning and end of the month to account for changes in transitory storage. These corrections were derived from the recession analysis. The "generated runoff" (Table 5) represents the actual streamflow generated from precipi-

TABLE 2.—COASTAL WINTER-FLOW INDEX, OCTOBER THROUGH MARCH, IN PERCENT OF BASE PERIOD AVERAGE, 1929-1950

Water Year (1)	Willamette River at Albany (2)	Wilson River near Tillamook (3)	Lewis River at Ariel (4)	Chehalis River near Grand Mound (5)	4-Station Average (6)	Recession Correction (7)	Coastal Flow Index[a] (CFI), % (8)
1959	115.0	107.0	129.0	120.0	118.0	+2.0	120.0
1958	136.0	104.0	114.0	97.0	113.0	-1.0	112.0
1957	123.0	85.0	106.0	98.0	103.0	+1.0	104.0
1956	208.0	165.0	159.0	191.0	181.0	+1.0	182.0
1955	77.0	90.0	80.0	93.0	85.0	+1.0	86.0
1954	153.0	129.0	129.0	149.0	150.0	-1.0	149.0
1953	123.0	103.0	112.0	99.0	109.0	0.0	109.0
1952	138.0	109.0	106.0	103.0	114.0	+1.0	115.0
1951	193.0	135.0	143.0	165.0	159.0	-0.5	158.5
1950	125.9	134.2	131.9	153.4	136.3	-0.2	136.1
1949	121.9	113.8	88.5	130.0	113.6	-0.4	113.2
1948	148.3	114.4	118.9	114.6	124.1	+0.2	124.3
1947	113.4	110.7	132.6	111.6	117.1	+0.9	118.0
1946	141.6	125.5	114.1	115.2	124.1	-0.5	123.6
1945	78.6	83.5	83.1	82.4	81.9	-0.5	81.4
1944	62.1	62.1	63.1	56.9	61.0	-1.3	59.7
1943	165.4	111.7	107.9	98.7	120.9	+2.7	123.6
1942	102.3	80.7	89.5	85.2	89.4	-1.8	87.6
1941	61.0	55.3	67.9	57.7	60.5	-2.4	58.1
1940	79.3	91.3	101.6	94.2	91.6	-0.7	90.9
1939	87.4	83.7	85.7	90.8	87.1	-0.5	86.6
1938	149.8	123.3	126.8	124.9	131.2	+0.3	131.5
1937	63.4	69.4	54.2	68.7	63.9	-0.4	63.5
1936	95.3	94.2	82.0	91.8	90.8	-0.4	90.4
1935	116.2	138.9	123.2	144.0	130.6	0.0	130.6
1934	81.9	147.0	201.9	149.4	145.0	+0.6	145.6
1933	108.1	126.1	113.6	139.9	121.9	+0.8	122.7
1932	114.7	119.4	111.5	118.9	116.1	+1.7	117.8
1931	47.9	56.6	65.5	64.2	58.6	+4.1	62.7
1930	73.4	83.0	71.9	57.3	71.4	-1.6	69.8
1929	61.5	75.4	63.4	51.1	62.8	-0.3	62.5
				1929-1959 Mean Index			107.6

a Column 6 + Column 7

tation during the month of April. The April correction index is equal to the generated runoff times the CFI.

Spring-Precipitation Index.—During the spring and summer months, the normal winter atmospheric flow pattern may be altered, so that the runoff from coastal streams may not adequately reflect the precipitation over the Columbia River Basin as a whole. The effect of spring-precipitation must

be evaluated as one of the factors affecting the total seasonal runoff. Therefore, a spring-precipitation index for the Columbia Basin as a whole was computed, using USWB and Canadian climatological stations having records for the period of study and presently reporting on the Columbia River Co-

TABLE 3.—WINTER RUNOFF OF THE COLUMBIA RIVER NEAR THE DALLES, OREG.[a]

Water Year (1)	October through March Runoff[b] (2)	Winter Runoff Correction Index[c] (3)
1959	41.48	49.78
1958	34.48	38.62
1957	35.81	37.24
1956	46.72	85.12
1955	30.93	26.60
1954	34.43	51.30
1953	31.67	34.52
1952	38.66	44.46
1951	51.43	81.52
1950	37.06	50.44
1949	34.67	39.25
1948	40.74	50.64
1947	40.16	47.39
1946	33.91	41.91
1945	27.45	22.94
1944	24.62	14.70
1943	35.34	43.68
1942	41.26	36.14
1941	31.26	18.16
1940	32.08	29.16
1939	28.02	24.27
1938	34.93	45.93
1937	20.01	12.71
1936	24.02	21.71
1935	33.31	43.50
1934	58.66	85.41
1933	31.17	38.26
1932	29.63	34.90
1931	23.07	14.46
1930	23.74	16.57
1929	25.60	16.00

[a] In 1,000,000 Acre Ft

[b] Corrected for storage in Kootenay, Hungry Horse, Flathead, Pend Oreille, Franklin D. Roosevelt, Coeur d'Alene, Chelan, and Brownlee Reservoirs

[c] Product, Coastal Winter-Flow Index (Table 2) times October through March Runoff (Divided by 100)

operative Hydrometeorological Reporting Network. There are sixteen such stations, the locations of which are shown on Fig. 1. Inspection of the geographical locations of the stations reveals that they are fairly well distributed with respect to basin coverage. The arithmetic sum of the precipitation in inches at all stations for the months of May and June for each year was com-

puted, and the average for each month was determined. These values are the spring-precipitation indexes for each of the months and are listed in Table 6.

TABLE 4.—WINTER TEMPERATURE INDEX[a]

Water Year (1)	October (2)	November (3)	February (4)	March (5)	4-Month Average, Degrees F (6)	Temp. Correction Index[c] (7)
1959	49.0	35.2	30.4	39.2	38.4	46.14
1958	45.5	35.3	38.2	38.9	39.5	44.21
1957	46.7	33.2	28.8	38.6	36.8	38.30
1956	47.9	27.5	22.8	36.5	33.7	61.29
1955	45.9	41.4	27.2	31.2	36.4	31.33
1954	49.9	40.5	33.1	36.2	39.9	59.49
1953	52.7	32.6	35.0	40.3	40.2	43.76
1952	46.1	35.0	30.3	36.5	37.0	42.52
1951	48.2	35.7	31.6	33.8	37.3	59.16
1950	44.3	39.6	29.7	37.0	37.6	51.24
1949	47.1	35.2	26.4	38.5	36.8	32.46
1948	50.0	34.7	28.0	36.0	37.2	46.21
1947	43.6	33.1	34.7	42.7	38.5	45.46
1946	51.6	36.1	33.7	41.7	40.8	50.40
1945	54.6	37.4	34.6	38.6	41.3	33.62
1944	51.1	38.6	32.6	37.0	39.8	23.78
1943	50.2	35.3	33.9	36.1	38.9	48.05
1942	48.1	39.4	30.7	39.8	39.5	34.60
1941	53.5	33.3	36.6	45.2	42.1	24.49
1940	48.3	38.8	36.9	46.5	42.6	38.75
1939	47.3	32.6	26.3	38.7	36.2	31.37
1938	51.9	39.0	30.4	38.9	40.0	52.67
1937	51.6	33.4	27.5	41.6	38.5	24.46
1936	47.3	33.1	17.6	38.1	34.0	30.76
1935	50.7	42.1	33.6	37.4	41.0	53.48
1934	50.8	39.7	37.8	45.4	43.4	63.19
1933	48.0	39.7	22.9	38.9	37.4	45.86
1932	48.8	33.0	28.3	37.6	36.9	43.53
1931	46.2	35.5	34.2	40.5	39.1	24.52
1930	49.9	35.2	37.1	41.0	40.8	28.48
1929	48.2	38.0	21.6	41.4	37.3	23.31
Mean	48.9	36.0	30.7	39.0	38.7	41.19

[a] 5-Station Average: Revelstoke, British Columbia; Missoula, Mont.; Spokane, Wash.; Yakima, Wash.; Boise, Idaho.

[b] Mean Monthly Temperature in Degrees Fahrenheit.

[c] Product, Coastal Winter-Flow Index (Table 2) times average temperature (divided by 100).

The total index is the sum of the May and June precipitation for each year, and the May-June correction index is the product of this total times the CFI.

ANALYSIS AND RESULTS

It is indeed fortunate that this index is feasible for one of the principal rivers of the United States, for which forecasting water supply is a prime

requisite for optimum use of multiple-purpose reservoir storage. Table 7 presents comparative area-elevation data for the Columbia River east of the Cascade and the coastal winter-flow index drainages. On the basis of the average for the four coastal winter-flow index drainages, a smaller percentage

TABLE 5.—APRIL COASTAL FLOW INDEX CHEHALIS RIVER NEAR GRAND MOUND, WASH.[a]

Water Year (1)	April Monthly Runoff, 1,000 Acre Ft (2)	Recession Correction, 1,000 Acre Ft (3)	Generated Runoff, 1,000 Acre Ft (4)	April Correction Index[b] 1,000 Acre Ft (5)
1959	250	-10	240	288.0
1958	230	+ 8	238	266.6
1957	160	-46	114	118.6
1956	200	-106	94	171.1
1955	330	-65	265	235.9
1954	210	-16	194	289.0
1953	100	-10	90	98.1
1952	110	-46	64	73.6
1951	110	-72	38	60.2
1950	230	-45	185	251.8
1949	90	-31	59	66.8
1948	200	- 2	198	246.1
1947	140	-22	118	139.2
1946	150	-22	128	157.6
1945	150	-32	118	96.1
1944	140	+ 3	143	85.4
1943	250	-98	152	187.9
1942	70	-18	52	45.6
1941	70	-19	51	29.6
1940	190	-99	91	82.7
1939	60	-37	23	19.9
1938	200	-43	157	206.5
1937	400	- 5	395	250.8
1936	90	-76	14	12.7
1935	120	-54	66	86.2
1934	90	-66	24	34.9
1933	40	-103	37	45.4
1932	260	-59	201	236.8
1931	310	-151	159	99.7
1930	90	-13	77	53.7
1929	260	-87	153	95.6
Mean, 1929-1959			127	133.3

[a] D. A. = 895 Sq Miles
[b] Product, Coastal Winter-Flow Index (Table 2) times generated runoff (divided by 100)

of area lies above 3,000 ft on the coastal index drainage areas than lies above 6,000 ft in the Columbia River Basin. In addition to the elevation differences, there are also differences in air-mass characteristics which affect the seasonal distribution of runoff between the basins. Also, when considering

runoff-producing areas, the lower elevation zones in the Columbia Basin principally comprise the arid Columbia and Snake River plains which produce virtually no runoff. As a result nearly all of the runoff in the Columbia Basin is generated from areas above 4,000 ft.

TABLE 6.—COLUMBIA RIVER BASIN SPRING-PRECIPITATION INDEX (MAY - JUNE)[a]

Water Year (1)	May (2)	June (3)	Spring Precipitation[b] Index, Inches (4)	May-June Correction Index[c] (5)
1959	1.99	1.31	3.30	3.960
1958	1.07	2.25	3.32	3.718
1957	2.77	1.57	4.34	4.514
1956	1.33	1.64	2.97	5.405
1955	1.40	1.55	2.95	2.537
1954	1.11	1.59	2.69	4.008
1953	1.86	1.84	3.70	4.033
1952	1.55	2.25	3.80	4.370
1951	1.20	1.20	2.40	3.804
1950	0.68	1.85	2.53	3.443
1949	1.39	0.76	2.15	2.434
1948	2.82	2.56	5.38	6.687
1947	0.83	2.44	3.27	3.859
1946	1.40	1.50	2.90	3.584
1945	1.93	1.38	3.31	2.634
1944	1.28	2.18	3.40	2.030
1943	1.00	1.92	2.92	3.609
1942	3.32	1.87	5.19	4.546
1941	2.13	2.31	4.44	2.580
1940	0.80	0.51	1.31	1.908
1939	1.16	1.57	2.73	2.364
1938	1.48	1.56	3.04	3.998
1937	0.69	1.74	2.43	1.543
1936	0.86	1.93	2.79	2.522
1935	0.89	0.98	1.87	2.442
1934	0.61	1.50	2.11	3.072
1933	1.67	0.98	2.65	3.252
1932	1.36	0.87	2.23	2.627
1931	0.55	1.44	1.99	1.248
1930	1.73	1.14	2.87	2.003
1929	0.78	1.50	2.28	1.425
Average	1.41	1.60	3.01	

[a] 16-Station Average, in inches
[b] May and June
[c] Product, Coastal Winter-Flow Index (Table 2) times May-June Precipitation Index (divided by 100)

On coastal index streams about 90% of the runoff is generated from areas below 4,000 ft in elevation. These differences are reflected in the average flow characteristics, as illustrated by the fact that about 28% of the annual runoff for Columbia River near The Dalles occurs in the period October

through March, while about 73% of the annual runoff is produced on the coastal tributaries for the same period. The difference between the two represents, principally, the differences in snow-water storage in the respective basins. An approximation of annual runoff amounts reveals that on the average, the total runoff west of the Cascade Range in Washington and Oregon (about 63,000 sq miles) is about 150,000,000 acre-ft, as compared with 140,000,000 acre-ft of runoff for the Columbia Basin east of the Cascades (237,000 sq miles).

Derivation of Forecasting Relationships.—The initial work on the coastal flow index method for forecasting Columbia River runoff[6] was based on a graphical statistical solution through use of a coaxial diagram.[9] One of the principal reasons for the choice of this method was that it could account for inter-relationships between the "primary" and "secondary" independent variables to be expressed in the forecasting relationship. For example, the effect on runoff of spring precipitation is a function of the winter snow accumulation as well as the magnitude of the spring precipitation itself.

TABLE 7

Location	Drainage Areas	Percent of Area Above Given Elevation			
		3000 ft	4000 ft	5000 ft	6000 ft
Columbia River near The Dalles	237,000	80	63	45	26
Willamette River at Albany, Oreg.	4,840	26	13	5	1
Lewis River at Ariel, Wash.	731	42	15	3	1
Wilson River near Tillamook, Oreg.	159	6	0	0	0
Chehalis River near Grand Mound, Wash.	895	1	0	0	0
Average for four coastal winterflow index stations		19	7	2	0.5

With the advent of the electronic digital computer, the solution of multiple linear regression equations became fast and relatively simple for statistical analysis. By combining certain independent variables as product functions, it is possible to account for variation in the dependent variables resulting from inter-actions between the independent variables. This technique was utilized by the Snow Investigation Unit in the multiple linear regression analysis of snowmelt rates, where there was known to be a physical law relating two independent variables as a product.

For the multiple linear regression analysis presented herein, the primary independent variable is the CFI, which indexes the total moisture input to the basin. Inasmuch as precipitation is primarily in the form of snow, the CFI is an index of the total snowpack accumulation. The "secondary" independent variables of winter runoff, winter temperature, April runoff, and May-June precipitation, are all dependent on the magnitude and areal extent of the snowpack in their effectiveness in producing runoff. Accordingly, the secondary independent variables are included as product functions with the CFI as a

[9] "Applied Hydrology," by R. K. Linsley, Jr., M. A. Kohler, and J. L. H. Paulhus, McGraw-Hill Book Co., Inc., New York, 1949, Appendix A.

realistic and rational combination so as to consider the variations in the runoff properly. After having arrived at this concept, it was found that correlations involving the "secondary" independent variables as the product functions with the CFI actually provided stronger correlations than when they were inserted as simple independent variables in the multiple correlations.

Multiple correlations based on this principle were computed for several combinations of independent variables for the period of record. The parameters were finally selected for use in the forecast equation on the basis of indexes which rationally account for the various phases of the water balance for the basin. They do not necessarily represent the "best fit" of historic data of all parameters investigated, which might be selected without due regard to physical explanation in terms of known hydrologic principles.

The forecast equation was computed by use of the "stepwise" multiple linear regression program[10] for the computer. All variables affecting runoff, including spring precipitation, were inserted in the regression analysis, in order to obtain the most realistic weighting of each of the variables. The equation so derived is equivalent to the forecast equation for conditions known as of July 1 of each year. Forecasts as of April 1, May 1, and June 1 were computed by using mean values of each of the variables affecting runoff after the date of the forecast. The basic forecast equation is

$$Y = 0.7428\ X_1 - 0.2836\ X_2 - 0.678\ X_3 + 0.0358\ X_4 + 6.50\ X_5 + 29.6 \quad .\ .\ .\ . \quad (1)$$

When Y is the April through September runoff for the Columbia River at The Dalles, in millions of acre-ft; X_1 is the coastal winter-flow index (CFI) in percent (Table 2); X_2 is the winter runoff correction index, in millions of acre-ft (Table 3); X_3 is the winter temperature correction index in degrees, F (Table 4); X_4 is the April generated Grand Mound runoff correction index, in thousands of acre-ft (Table 5); and X_5 is the May-June precipitation correction index, in inches (Table 6). Table 8a lists the values of each of the parameters, together with the forecast and observed runoff, for each of the years of study, based on the solution of Eq. 1. Results of this correlation present a measure of the effectiveness of all parameters used in estimating seasonal runoff, and thereby represent forecasts that would have been made as of July 1 of each year. Tables 8b, 8c, and 8d show similar data for forecasts which would be made from data available as of June 1, May 1, and April 1 of each year, respectively.

The mathematically derived forecast equation can be shown in graphical form to illustrate the relative magnitude of individual variables in accounting for runoff variance, the ranges of each of the variables for the period of record, and the effect of the product function in weighting the "secondary" independent variables. Fig. 2(a) is the graphical solution of Eq. 1, for the winter components of the forecast equation, for which the CFI is plotted as the abscissa and April through September runoff components are plotted as ordinate values. The parameters of October through March runoff and the winter temperature index are shown above and below the central base line. Ordinate values above the central base line represent the evaluation of the

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10 "Operating Instruction for Use of the Stepwise Regression Procedure on the IBM 650, File 4260," by M. A. Efroymson and O. Schriker, Esso Research and Engrg. Co., Process Research Div., Linden, N. J.

TABLE 8.—FORECAST DATA FOR APRIL THROUGH SEPTEMBER RUNOFF COLUMBIA RIVER NEAR THE DALLES, OREG.[a]

Water Year	X_1 Coastal Flow Index, (CFI)[b]	X_2 CFI Winter Runoff at The Dalles[c]	X_3 CFI Winter Temperature Index[d]	X_4 CFI April Generated Grand Mound Runoff[e]	X_5 CFI May-June Precipitation Index[f]	April through September Runoff, Columbia River near The Dalles, in 10^6 Ac Ft[g]		
						Forecast	Observed	Deviation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				(a) FORECAST AS OF JULY 1				
1959	120.0	49.78	46.14	288.0	3.960	109.4	112.8	-3.4
1958	112.0	38.62	44.21	266.6	3.718	105.6	99.0	6.6
1957	104.0	37.24	38.30	118.6	4.514	103.9	104.5	- .6
1956	182.0	85.12	61.29	171.1	5.405	140.4	131.7	8.7
1955	86.0	26.60	31.33	235.9	2.537	89.6	99.5	-9.9
1954	149.0	51.30	59.49	289.0	4.008	121.8	118.0	3.8
1953	109.0	34.52	43.76	98.1	4.033	100.8	99.8	1.0
1952	115.0	44.46	42.52	73.6	4.370	104.6	107.7	-3.1
1951	158.5	81.52	59.16	60.2	3.804	111.0	112.6	-1.6
1950	136.1	50.44	51.24	251.8	3.443	113.0	120.3	-7.3
1949	113.2	39.25	32.46	66.8	2.434	98.8	95.7	3.1
1948	124.3	50.64	46.21	246.1	6.687	128.5	130.4	-1.9
1947	118.0	47.39	45.46	139.2	3.859	103.1	100.3	2.8
1946	123.6	41.91	50.40	157.6	3.584	104.3	108.1	-3.8
1945	81.4	22.94	33.62	96.1	2.634	81.3	81.5	- .2
1944	59.7	14.70	23.78	85.4	2.030	69.9	61.8	8.1
1943	213.6	43.68	48.05	187.9	3.609	106.6	115.0	-8.4
1942	87.6	36.14	34.60	45.6	4.546	92.1	90.2	1.9
1941	58.1	18.16	24.49	29.6	2.580	68.8	69.0	- .2
1940	90.9	29.16	38.75	82.7	1.908	77.9	77.1	.8
1939	86.6	24.27	31.37	19.9	2.364	81.8	80.5	1.3
1938	131.5	45.93	52.67	206.5	3.998	111.9	102.5	9.4
1937	63.5	12.71	24.46	250.8	1.543	75.6	72.5	3.1
1936	90.4	21.71	30.76	12.7	2.522	86.6	90.5	-3.9
1935	130.6	43.50	53.48	86.2	2.442	97.0	89.9	7.1
1934	145.6	85.41	63.19	34.9	3.072	91.9	94.8	-2.9
1933	122.7	38.26	45.86	45.4	3.252	101.6	111.8	-10.3
1932	117.8	34.90	43.53	236.8	2.627	103.2	105.6	-2.4
1931	62.7	14.46	24.52	99.7	1.248	67.1	65.0	2.1
1930	69.8	16.57	28.48	53.7	2.003	72.4	71.0	1.4
1929	62.5	16.00	23.31	95.6	1.425	68.4	69.8	-1.4

TABLE 8.–*CONTINUED*

(1)	(2)[b]	(3)[c]	(4)[d]	(5)[e]	(6)[h]	(7)[g]	(8)[g]	(9)[g]
(b) FORECAST AS OF JUNE 1								
1959	120.0	49.78	46.14	288.0	4.308	111.6	112.8	-1.2
1958	112.0	38.62	44.21	266.6	2.990	100.8	99.0	1.8
1957	104.0	37.24	38.30	118.6	4.545	104.1	104.5	- .4
1956	182.0	85.12	61.29	171.1	5.333	139.9	131.7	8.2
1955	86.0	26.60	31.33	235.9	2.580	89.9	99.5	-9.6
1954	149.0	51.30	59.49	289.0	4.037	122.0	118.0	4.0
1953	109.0	34.52	43.76	98.1	3.771	99.1	99.8	- .7
1952	115.0	44.46	42.52	73.6	3.622	99.8	107.7	-7.9
1951	158.5	81.52	59.16	60.2	4.438	115.1	112.6	2.5
1950	136.1	50.44	51.24	251.8	3.103	110.8	120.3	-9.5
1949	113.2	39.25	32.46	66.8	3.385	104.9	95.7	9.2
1948	124.3	50.64	46.21	246.1	5.494	120.8	130.4	-9.6
1947	118.0	47.39	45.46	139.2	2.867	96.6	100.3	-3.7
1946	123.6	41.91	50.40	157.6	3.708	105.1	108.1	-3.0
1945	81.4	22.94	33.62	96.1	2.873	82.9	81.5	1.4
1944	59.7	14.70	23.78	85.4	1.719	67.9	61.8	6.1
1943	123.6	43.68	48.05	187.9	3.214	104.0	115.0	-10.9
1942	87.6	36.14	34.60	45.6	4.310	90.6	90.2	.4
1941	58.1	18.16	24.49	29.6	2.167	66.1	69.0	-2.9
1940	90.9	29.16	38.75	82.7	2.182	79.7	77.1	2.6
1939	86.6	24.27	31.37	19.9	2.390	82.0	80.5	1.5
1938	131.5	45.93	52.67	206.5	4.050	112.2	102.5	9.8
1937	63.5	12.71	24.46	250.8	1.454	75.0	72.5	2.5
1936	90.4	21.71	30.76	12.7	2.224	84.6	90.5	-5.9
1935	130.6	43.50	53.48	86.2	3.252	102.2	89.9	12.3
1934	145.6	85.41	63.19	34.9	3.218	92.8	94.8	-2.0
1933	122.7	38.26	45.86	45.4	4.012	106.5	111.8	-5.3
1932	117.8	34.90	43.53	236.8	3.487	108.8	105.6	3.2
1931	62.7	14.46	24.52	99.7	1.348	67.8	65.0	2.8
1930	69.8	16.57	28.48	53.7	2.324	74.5	71.0	3.5
1929	62.5	16.00	23.31	95.6	1.487	68.8	69.8	-1.0

TABLE 8.–*CONTINUED*

(1)	(2)[b]	(3)[c]	(4)[d]	(5)[e]	(6)[i]	(7)[g]	(8)[g]	(9)[g]
(c) FORECAST AS OF MAY 1								
1959	120.0	49.78	46.14	288.0	3.610	107.1	112.8	-5.7
1958	112.0	38.62	44.21	266.6	3.369	103.3	99.0	4.3
1957	104.0	37.24	38.30	118.6	3.128	94.9	104.5	-9.6
1956	182.0	85.12	61.29	171.1	5.475	140.8	131.7	9.1
1955	86.0	26.60	31.33	235.9	2.587	90.0	99.5	-9.5
1954	149.0	51.30	59.49	289.0	4.482	124.9	118.0	6.9
1953	109.0	34.52	43.76	98.1	3.279	95.9	99.8	-3.9
1952	115.0	44.46	42.52	73.6	3.459	98.7	107.7	-9.0
1951	158.5	81.52	59.16	60.2	4.768	117.2	112.6	4.6
1950	136.1	50.44	51.24	251.8	4.094	117.3	120.3	-3.0
1949	113.2	39.25	32.46	66.8	3.405	105.1	95.7	9.4
1948	124.3	50.64	46.21	246.1	3.739	109.4	130.4	-21.0
1947	118.0	47.39	45.46	139.2	3.549	101.0	100.3	- .7
1946	123.6	41.91	50.40	157.6	3.718	105.2	108.1	-2.9
1945	81.4	22.94	33.62	96.1	2.449	80.1	81.5	-1.4
1944	59.7	14.70	23.78	85.4	1.796	68.4	61.8	6.6
1943	123.6	43.68	48.05	187.9	3.718	107.3	115.0	-7.7
1942	87.6	36.14	34.60	45.6	2.635	79.7	90.2	-10.5
1941	58.1	18.16	24.49	29.6	1.748	63.4	69.0	-5.6
1940	90.9	29.16	38.75	82.7	2.734	83.3	77.1	6.2
1939	86.6	24.27	31.37	19.9	2.605	83.4	80.5	2.9
1938	131.5	45.93	52.67	206.5	3.956	111.6	102.5	9.1
1937	63.5	12.71	24.46	250.8	1.910	78.0	72.5	5.5
1936	90.4	21.71	30.76	12.7	2.719	87.9	90.5	-2.6
1935	130.6	43.50	53.48	86.2	3.928	106.6	89.9	16.7
1934	145.6	85.41	63.19	34.9	4.380	100.4	94.8	5.6
1933	122.7	38.26	45.86	45.4	3.691	104.4	111.8	-7.4
1932	117.8	34.90	43.53	236.8	3.543	109.2	105.6	3.6
1931	62.7	14.46	24.52	99.7	1.886	71.3	65.0	6.3
1930	69.8	16.57	28.48	53.7	2.100	73.0	71.0	2.0
1929	62.5	16.00	23.31	95.6	1.880	71.3	69.8	1.5

TABLE 8.–*CONTINUED*

(1)	(2)[b]	(3)[c]	(4)[d]	(5)[j]	(6)[i]	(7)[g]	(8)[g]	(9)[g]
(d) FORECASTS AS OF APRIL 1								
1959	120.0	49.78	46.14	152.4	3.610	102.3	112.8	10.5
1958	112.0	38.62	44.21	142.2	3.369	98.9	99.0	- .1
1957	104.0	37.24	38.30	132.1	3.128	95.4	104.5	-9.1
1956	182.0	85.12	61.29	231.1	5.475	143.0	131.7	11.3
1955	86.0	26.60	31.33	109.2	2.587	85.4	99.5	-14.1
1954	149.0	51.30	59.49	189.2	4.482	121.3	118.0	3.3
1953	109.0	34.52	43.76	138.4	3.279	97.4	99.8	-2.4
1952	115.0	44.46	42.52	146.0	3.459	101.3	107.7	-6.4
1951	158.5	81.52	59.16	201.3	4.768	122.3	112.6	9.7
1950	136.1	50.44	51.24	172.8	4.094	114.4	120.3	-5.9
1949	113.2	39.25	32.46	143.8	3.405	107.8	95.7	12.1
1948	124.3	50.64	46.21	157.9	3.739	106.2	130.4	-24.2
1947	118.0	47.39	45.46	149.9	3.549	101.4	100.3	1.1
1946	123.6	41.91	50.40	157.0	3.718	105.1	108.1	-3.0
1945	81.4	22.94	33.62	103.4	2.449	80.4	81.5	-1.1
1944	59.7	14.70	23.78	75.8	1.796	68.0	61.8	6.2
1943	123.6	43.68	48.05	157.0	3.718	106.2	115.0	-8.8
1942	87.6	36.14	34.60	111.3	2.635	82.1	90.2	-8.1
1941	58.1	18.16	24.49	73.8	1.748	65.0	69.0	-4.0
1940	90.9	29.16	38.75	115.4	2.734	84.5	77.1	7.4
1939	86.6	24.27	31.37	110.0	2.605	86.6	80.5	6.1
1938	131.5	45.93	52.67	161.7	3.956	110.0	102.5	7.5
1937	63.5	12.71	24.46	80.6	1.910	71.9	72.5	-0.6
1936	90.4	21.71	30.76	114.8	2.719	91.5	90.5	1.0
1935	130.6	43.50	53.48	165.9	3.928	109.5	89.9	19.6
1934	145.6	85.41	63.19	184.9	4.380	105.8	94.8	11.0
1933	122.7	38.26	45.86	155.9	3.691	108.4	111.8	-3.4
1932	117.8	34.90	43.53	149.6	3.543	106.1	105.6	0.5
1931	62.7	14.46	24.52	79.6	1.886	70.6	65.0	5.6
1930	69.8	16.57	28.48	88.6	2.100	74.3	71.0	3.3
1929	62.5	16.00	23.31	79.4	1.880	70.7	69.8	9

[a] $Y = 0.7428X_1 - 0.2836X_2 - 0.678X_3 + 0.0358X_4 + 6.50X_5 + 29.6$ [b] Table 5 [c] Table 6 [d] Table 7 [j] Mean April generated runoff, Chehalis River near Grand Mound (127,000 acre-ft) times CFI [i] Mean May and June precipitation Index (3.01 in.), times CFI [g] Corrected for storage in Kootenay, Hungry Horse, Flathead, Pend Oreille, Franklin D. Roosevelt, Coeur d'Alene, Chelan, and Brownlee Reservoirs [h] Observed May precipitation index plus mean June precipitation index (1.60 in.) times CFI.

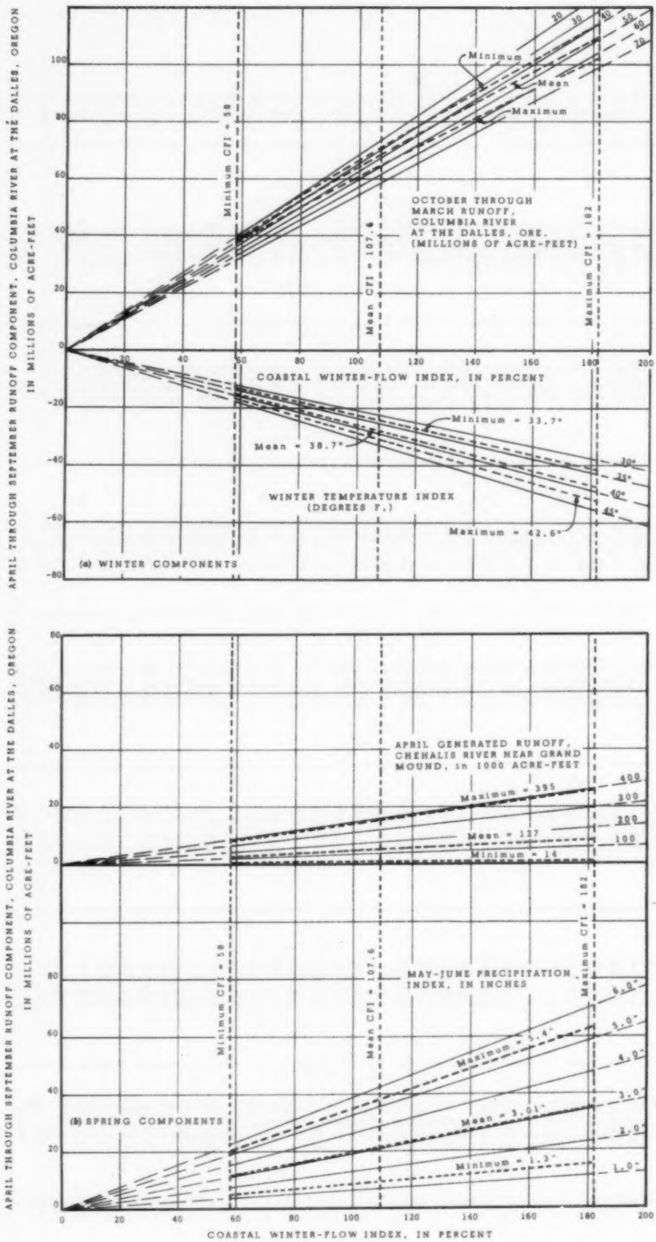


FIG. 2(a),(b).—GRAPHICAL SOLUTION OF FORECAST EQUATION

first two terms in Eq. 1, as a function of CFI and October through March runoff. Ordinate values below the base line for specified conditions of CFI and winter temperature index provide the solution to the third term in Eq. 1. Fig. 2(b) shows similar graphs for the effects of the indexes of April runoff for Chehalis River near Grand Mound and May-June precipitation, combined as product functions of CFI.

The means' values and extremes of each of the index values for the 31-yr period, 1929 through 1959, are indicated on the graphs. The forecast runoff for a specific set of conditions can be obtained by the algebraic sum of the ordinates of each of the four components shown in Figs. 2(a) and 2(b), plus the regression constant, 29.6 (all in unit of milliolns of acre-ft). In application, of course, it is simpler to solve Eq. 1 arithmetically, but the primary purpose of the graphical solution is to illustrate the effect of the product functions.

If the secondary independent variables were entered into the regression analysis as simple variables, rather than as product functions, the parameter lines in Figs. 2(a) and 2(b) would be parallel. It is reasonable to assume, for these particular indexes, that their effect on runoff is increased with increasing amounts of winter snow accumulation, mainly as the result of the greater snow covered area that would generally prevail. Accordingly, the diverging shape of the parameters as shown in Figs. 2(a) and 2(b), resulting from the product functions, is a more reasonable solution than using simple unrelated independent variables.

TABLE 9

Date of Forecast (1)	Average Error, 10^6 Acre Feet (2)	Standard Error of Estimate 10^6 Acre Feet (3)	Correlation Coefficient, R (4)	Coefficient of Determination D (= R^2) (5)
April 1	6.72	9.34	0.880	0.775
May 1	6.48	8.48	0.906	0.821
June 1	4.69	6.47	0.947	0.898
July 1	3.94	5.55	0.960	0.924

Summary of Results.—The comparisons of forecast versus actual runoff listed on Table 8, are plotted on Figs. 3. Table 9 lists statistical comparisons of April through September runoff forecasts which would have been made, as of April 1 through July 1, based on solution of the forecast equation for the 31-yr period, 1929 through 1959. Early season forecasts are based on assumed mean values of runoff index parameters for those variables which would enter the equation subsequent to the date of forecast. The mean April through September runoff for the period is 96,420,000 acre-ft, and the standard deviation of the runoff is 19,030,000 acre-ft. The standard errors of estimate and the correlation coefficients shown above were computed on the basis of following numbers of degrees of freedom:

April 1	27	(n - 4)
May 1	26	(n - 5)
June 1	$25\frac{1}{2}$	(n - $5\frac{1}{2}$)
July 1	25	(n - 6)

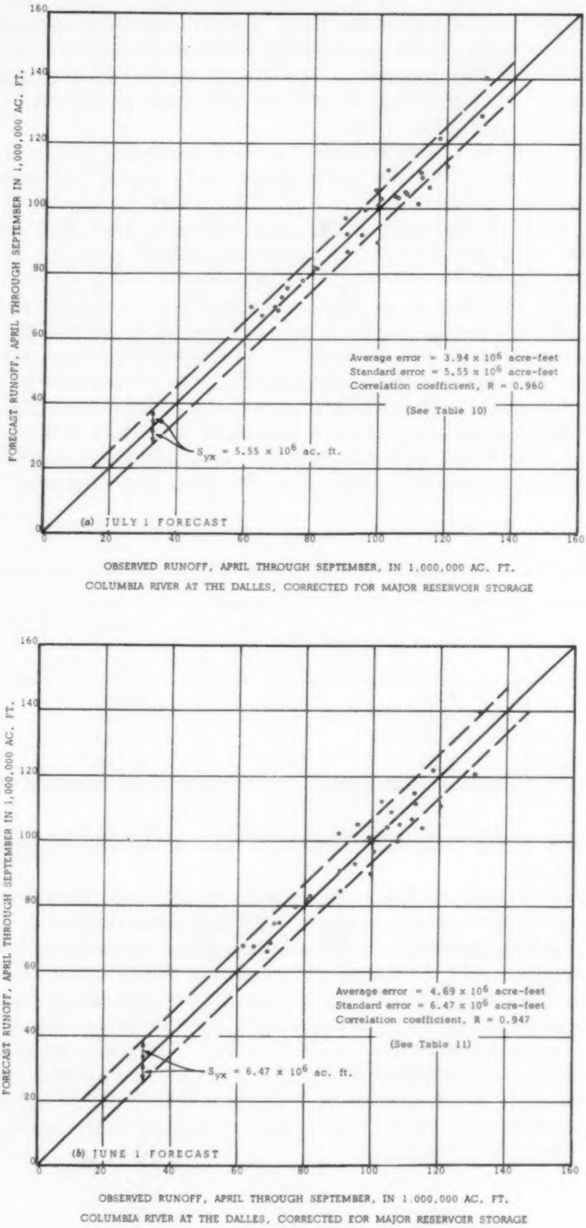
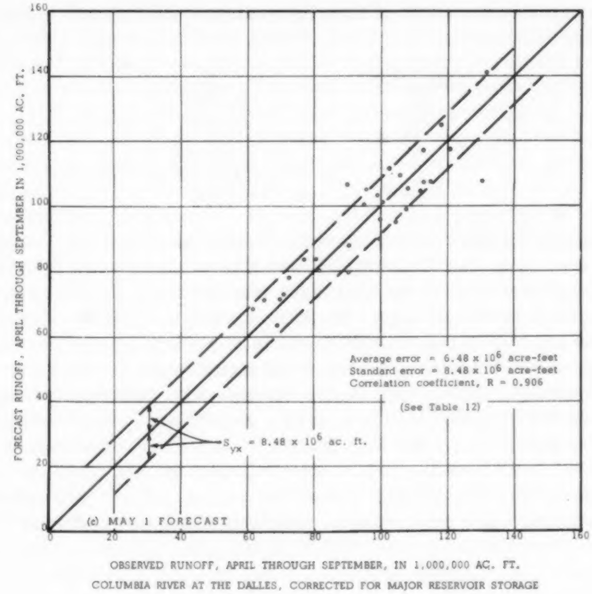


FIG. 3(a),(b).—FORECAST VS. ACTUAL RUNOFF



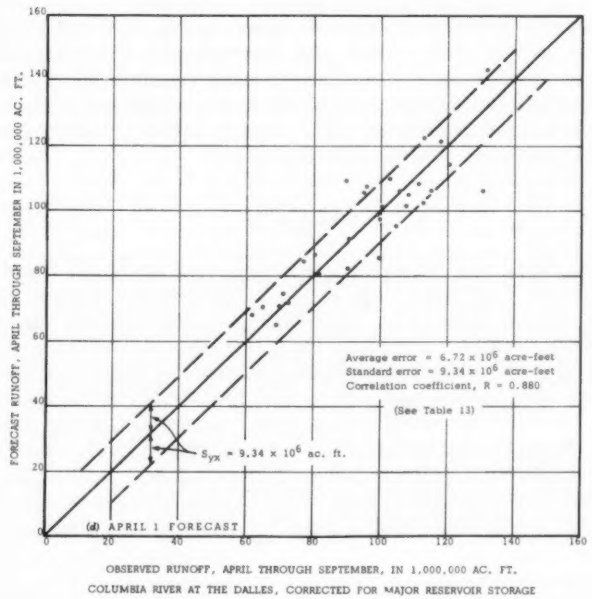


FIG. 3(c),(d).—FORECAST VS. ACTUAL RUNOFF

The standard errors for each of the regression coefficients for the variables in Eq. 1 are as follows:

Variable	S_b
X_1	0.1345
X_2	0.1538
X_3	0.3100
X_4	0.0132
X_5	1.150

Comparison of Results With Other Index Procedures.—The prime forecast of seasonal runoff in the Columbia River Basin has been April 1, partly because snow survey data as of that date has the most general coverage, but also because that date represents the time at which a fairly reliable forecast of streamflow may be determined and yet is early enough to take into account operational problems that may be incurred on the basis of the forecast runoff. In 1954 the technical staff of the Water Management Subcommittee, Columbia Basin Inter-agency Committee, prepared a report for review and comparison of forecasting procedures for The Dalles,[11] in which comparative measures of accuracy of four procedures in general use at that time for forecasting runoff in the Columbia Basin, based on April 1 forecasts are shown.[11a]

The procedures were developed by four United States government agencies, including the Corps of Engineers, USWB, Soil Conservation Service, and USGS. They were based primarily on snow survey or winter precipitation data, but with minor adjustments for other runoff factors. Inasmuch as they were developed for different forecast periods, a consistent measure of reliability was the standard error of estimate, expressed in percent of the average runoff for the forecast period. These values ranged from 11.0% to 13.2% for April 1 forecasts.[11a] For the CFI method, the standard error of 9,340,000 acre-ft, in terms of the average April through September runoff of 96,420,000 acre-ft, is 9.7%. The correlation coefficients of the four procedures ranged from 0.71 to 0.88, based on April 1 forecasts. The coastal winter flow index method has a correlation coefficient of 0.88, as of April 1.

COMMENTS

The derivation of this procedure of forecasting snowmelt runoff from the Columbia River at The Dalles is based on the geographic location of the basin with respect to the large-scale air-mass and atmospheric flow conditions during the winter season, as well as the differences of the topographic character, principally with respect to elevation, of runoff-producing areas, between the Columbia Basin and the adjacent coastal regions of the Pacific Northwest. Because the method is based on an index procedure, some consideration should be given to the various indexes presently used for forecasting runoff. In general, precipitation indexes, either from rain gage or snow course water-equivalent measurements, are based on samples which measure only an infinitesimal part of the total water supply for the Columbia Basin.

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11 "Review of Procedures for Forecast Seasonal Runoff of Columbia River near The Dallas, Oregon," Columbia Basin Inter-Agency Committee, Water Mgt. Subcommittee, Portland, Oreg., 1954. (a) Table 9, p. 49.

In the mountains of the west, where there is great areal variability of precipitation, the relation between gaged and actual precipitation amounts may vary from year to year as the result of meteorologic factors which, for the most part, are too complex for a rational analysis by existing techniques and intensity of the gage network. As an index, then, precipitation measurements cannot account for the random variability caused by inherent deficiencies of measurements by gages or snow courses, as well as the variability caused by the character of individual storms. The coastal winter-flow index, though having limitations, possesses certain advantages over gaged precipitation measurements. It samples a much larger proportion of the total water supply, as illustrated by the fact that in the Columbia Basin, the ratio is about one part in 14 in terms of runoff. The accuracy in the measurement of streamflow is an inherent limitation, but such deficiencies also apply to other index procedures for forecasting runoff. The principal limitation is the variability from year to year of the average air-mass characteristics, and the average atmospheric circulation pattern during times of significant precipitation.

The winter-temperature index and Columbia River winter runoff account for part of the variability caused by the air-mass differences. They occur from year to year. As indicated by the comparison of forecast errors, the coastal winter-flow index accounts for the variability in runoff as well as or better than precipitation indexes as measured at presently operated rain gages or snow courses, for the Columbia River Basin as a whole.

The effects of spring precipitation are accounted for in this procedure by the April generated runoff index for the Chehalis River near Grand Mound, and the May-June precipitation index, based on a 16-station mean. Use of statistical techniques for weighting individual indexes of runoff should be predicated on inclusion of all variables in the multiple regression analysis. By correlating runoff directly with just those values of the variables known as of April 1, it is possible to obtain an apparently better correlation for April 1 forecasts than using the correlation for all variables, and then computing forecasts for April 1, using mean spring precipitation. Omitting the spring precipitation effects in the derivation of April 1 forecasts is unrealistic and simply forces a best fit of the historic data with incomplete information. It does not necessarily provide the best weighting of individual winter indexes.

In the particular case of the winter indexes of runoff for the coastal winter-flow index method, correlating them directly with April through September runoff provides a standard error of estimate of 9,190,000 acre-ft, compared with 9,340,000 acre-ft for April 1 forecasts, obtained from the general forecast equation and using mean values of spring precipitation indexes. The apparent reduction in the standard error of estimates is fallacious. Similarly, direct correlation of indexes known on May 1 of each year would provide a standard error of estimate of 8,220,000 acre-ft, compared with 8,480,000 acre-ft for the forecast equation.

The use of product functions in multiple linear regression analysis of seasonal runoff variance may have application to forecast procedures based on snow course or precipitation gage indexes. Product functions should be used only where there is known to be a physical process relating two independent variables as a product.

Application to Other Basins.—Since this procedure deals with largescale meteorologic phenomena, its application is intended primarily for large basin areas, rather than smaller tributary areas. However, where there are

significant elevation differences between basin areas, it is believed that winter low-elevation runoff may be used to evaluate water stored in the snow-pack in the middle latitudes on basins smaller than the Columbia River provided that the index runoff area is well centered in the air stream dominating the basin area.

CONCLUSIONS

The coastal winter flow index, in conjunction with other runoff parameters, provides a basis for forecasting seasonal (April through September) runoff volume for the Columbia River at The Dalles. The historical streamflow and climatological records used in the correlations have a sufficient period of record (31 yr), considering the number of variables used (six), to insure a reasonably sound statistical analysis. The use of the "stepwise" multiple linear regression program for the electronic digital computer expedited the correlation analysis used in developing the basic forecast equation. The basic approach of correlating runoff (the dependent variable) with indexes of all of the known significant factors affecting runoff (the independent variables), is considered to be hydrologically sound. Apparent improvements in fit of historic data by direct correlations with runoff of variables known as of the date of forecast (that is, April 1) are considered to be fallacious.

The use of product functions of the primary and secondary independent variables improved the forecast relationship, in accounting for runoff variance. Product functions are believed to be valid in cases where the effectiveness of one independent variable is known to increase with increasing values of a second independent variable. Comparisons of forecasting methods for Columbia River at The Dalles, developed by government agencies for operational use in the Columbia River Basin, have been made by the technical staff of the Water Management Subcommittee, Columbia Basin Inter-Agency Committee. Use of these comparisons shows that the coastal winter flow index method is as accurate with regard to fit of historical data as other procedures based primarily upon winter precipitation or snow course data for forecasts made as of April 1.

ACKNOWLEDGMENT

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DISCUSSION

EFFECT OF AQUIFER TURBULENCE ON WELL DRAWDOWN[a]

Closure by Joe L. Mogg

JOE L. MOGG.[1]—Mr. Philip has called attention to a concept that was new to the author. He pointed out that inertial forces, rather than the inception of turbulence, initially caused the head loss to vary with an exponent of the velocity that was greater than one.

Mr. Philip's excellent and numerous references, plus some others were reviewed by the author. G. Schneebeli[2] describes experiments that also show that the head loss begins to vary with an exponent of the velocity greater than one after a critical Reynolds number is reached. However, dye introduced in the flow, for the purpose of identifying streamlines, showed that the flow was laminar and not turbulent in this non-linear region. Turbulence was evident at higher Reynolds numbers when the dye became readily dispersed. Thus, these experiments show that there is a broad region of flow between laminar-linear flow and turbulent-non-linear flow that is laminar, but not linear. The cause of the non-linearity could well be due to the inertial forces.

The conclusion is, then, that turbulence seldom occurs in the vicinity of a pumping well because most of the non-linear flow is actually laminar-non-linear flow.

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[a] November, 1959, by Joe L. Mogg (Proc. Paper 2265).

[1] Field Engr., Edward E. Johnson, Inc., St. Paul, Minn.

[2] "Experiments on the range of validity of Darcy's law and the appearance of turbulence in a filtering flow," by G. Schneebeli, La Houille Blanche, March, April, 1955, No. 2, p. 141.

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NEW APPROACH TO LOCAL FLOOD PROBLEMS[a]

Closure by Herbert D. Vogel

HERBERT D. VOGEL,[5] F. ASCE.—The observation by Bernard L. Golding, M. ASCE, to the effect that communities may make their own observations and determinations through local engineering forces and consulting engineers, is sound in principle. The primary need is to obtain adequate data from whatever sources may be available and make the best possible use of it.

Legislation has been enacted (1960) by the Congress to permit the United States Army, Corps of Engineers to carry out studies of this kind in watersheds and on rivers and tributaries under their jurisdiction. This provides a tool for the use of communities throughout the entire country and should encourage the approach to solution of flood problems that has been indicated.

However the studies may be made, it is a certainty that results will be of benefit to individual communities during years of growth and that savings of great magnitude will be made by the Federal Government in the future.

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[a] January, 1960, by Herbert D. Vogel (Proc. Paper 2336).
[5] Chmn., Tenn. Valley Authority, Knoxville, Tenn.

THE FOURTH ROOT n-f DIAGRAM[a]

Closure by T. Blench

T. BLENCH,[39] F. ASCE.—It is gratifying to find that the comments on an ostensibly practical paper have touched on so many of the fundamental points that occasioned it. However, if the most fundamental issue of all is not to be avoided in reply, as it could have been had comments concerned only details of the diagram and its use, the writer must confess to a third "object in mind" to add to the two stated by Gerald Lacey, discuss its reason, and then answer along lines that include detailed replies automatically. The third object in mind was to present practicing engineers with the means to free themselves from the trammels of the pseudo-science of engineering fluid mechanics so that they can develop knowledge of pipe flow in terms of their own data and the logic that they acquire in the exercise of their profession. The context and the references were intended to aid this object, whereas the diagram provided the means for co-ordinating data. The reason for the object is that the writer is one of those who deplores the long neglect of disciplined instruction in basic science and mathematics in North American schools, continued into college engineering curricula. In his opinion the neglect has been responsible for the displacement of the applied science of hydraulics by so-called fluid mechanics necessarily based on evasions, part-truths, dogmatism, and so forth. From such a subject no advance in fundamental understanding is possible; in fact it misdirects research. For example, to get away from pipes, the engineering student is taught a Bernouilli equation that allows for friction by pretending that pressure is the same in all directions about a point, omitting the velocity head of fluctuating motion, often ignoring the kinetic energy correction factor, and lumping all errors and omissions into a "loss of head." We may imagine the wasted effort if he were asked to conduct fundamental research into the hydraulic jump on a 45° spillway chute at a place at which velocity had become terminal. Here the shear stress on the floor equals the direct stress and one principal stress is zero, the shear stress along the floor in the jump is high, the kinetic energy and momentum correction factors and the turbulent velocity head at the control sections all merit attention. If he had no fluid mechanics training at all, but good fundamental dynamics and physics, his chances of success would be reasonable because he would know how to state his problem in terms of its relevant factors, translate it into the language of mathematics as necessary, then decide the extent to which it should be tackled inductively or deductively with or without expert aid. He could also research successfully if equipped to study hydraulics as presented

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[a] January 1960, by T. Blench (Proc. Paper 2340).

[39] Prof., Dept. of Civ. Engrg., Univ. of Alberta, Edmonton, Alberta, Canada.

in texts[40,41,42] that presume an adequate foundation of mathematics and physics, or draw attention to assumptions, idealizations and limitations and do not avoid an argument by "it can be proved that" or "according to so-and-so."

To avoid unintended offence to textbook writers the author states that he places blame squarely on neglected basic science education, has committed errors because of it, has experienced the difficulty of teaching inadequately prepared students, and cannot imagine a publisher accepting a non-conforming college book. He knows there is a changing attitude to education and that at least some colleges, including his own, are making desirable innovations.

Because of the preceding opinions an attempt will now be made to answer specific points in an informal manner that will distinguish definite knowledge from various degrees of conjecture.

The practical problem may be stated as "A Newtonian liquid flows under conditions of uniform dissipation of head (energy per unit weight) per unit length in a circular pipe of given material. Derive a formula to express the mean speed of flow, U, in terms of S, ρ, ν, and d and the boundary properties."

An omniscient being, OB, if asked to solve and explain, would probably start from the laws of intra and inter molecular action and combine them with a relativistic form of Newton's second law. These would show that the motion would not be steady, in general, but the flow would "roll itself up" into eddies whose statistical mechanics could be worked out so that the probability of a specified velocity at any point during any prescribed time interval could be deduced. Thence the mean behavior at every point could be determined and finally the mean velocity of the section in a form

$$U = \text{fn}\,(g\,S,\ d,\ \nu,\ \text{"roughness"}) \qquad (22)$$

from which ρ is missing, for dimensional reasons, and in which OB would have to explain how to measure "roughness." His answer would be the dynamically correct one as it would have been deduced correctly from unassailable premises in terms of undisputable laws. An ordinary hydraulics investigator, HI, of the classic school would admit he did not possess OB's omniscience so would proceed like his brother physicists who also had to unravel the mysteries of "fluids" (for example, electricity, heat) whose internal mechanism was not observable. That is, he would correlate measureable quantities covering the whole available range, and test formulas fitting the results to see if they suggested simple relations among dynamical entities - force, rate of work, and so forth. His knowledge of general physics would cause him to believe that if he were lucky enough to hit a formula identical with OB's true answer it would be simple and would certainly express some simple relation among dynamical entities. Such a relation might be reducible to minimal type, analogous to "light travels so as to make the time of transit a minimum," "free flow over a weir occurs so that the energy of flow is a minimum at a critical section . . . ." Although he would hope to discover the dynamically correct formula by his indirect approach he would expect the odds to be against him, particularly if the measureable quantities were not well defined or did not cover an adequate range. As a scientist he would not convince himself that he had made a discovery until he had checked it against

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40 "Fluid Dynamics," by L. Prandtl, Blackie & Son, 1952, pp. 125-130.

41 "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936.

42 "Fluid Mechanics and Heat Transfer," by J. M. Kay, Cambridge Univ. Press, New York, N. Y., 1957.

all available information and verified that it contained no physical contradictions. Then he might allow himself to state that he had found a relation that seemed to have, or suggest, certain dynamical meanings within a specified range of variable. The relative newcomer on the North American scene, the enthusiastic fluid mechanics investigator, EFMI, does not have HI's inhibitions but feels free, on occasions, to conjure formulas from the air, consider physical absurdities in them to be unimportant so long as the answer looks practically good, produce only enough data to justify his assumptions and extrapolate with happy abandon.

HI has developed two main lines of attack. One is to regard the flow formula as an equation between boundary resistance and driving force justifying the plotting of non-dimensional shear stress, f, against the Reynold's Number ($V\ d/\nu$) for various values of an unmeasureable relative roughness. This leads to the standard friction factor diagram and the discovery of different phases of turbulent flow. The practical and basic value of this diagram is that it is plotted from readily measureable variables, except for "roughness." So, if data can be separated into groups of approximately constant roughnesses, it gives a means of defining roughness in terms of whatever formula HI likes to try as his current guess at OB's and, of course, it shows him the range of conditions over which "roughness" is relevant to the flow. Then, if he finds that facts justify describing roughness by a "roughness height" e in the rough boundary range, he can define e according to Eq. 4 if he favors Manning, or by Eq. 5 if he favors the fourth root type of formula, or by the logarithmic formula that EFMI likes, or by any reasonably fitting formula; whatever formula he likes can be used, after algebraic conversion as with Eqs. 2 and 3, to draw lines of equal relative roughness on the friction factor diagram in the proper zone and to label them with appropriate e values. For the purely practical man, any system of e's is good enough provided the formula on which it is based is a good practical fit, for fixed e, in the range of data of interest to him; but, of course, HI will never be satisfied till he thinks he has hit OB's dynamically correct formula that would fit the full range of its dynamical applicability. The second line of attack follows logically from the first because the phases of turbulent flow shown by the f diagram direct attention immediately to the question whether they are associated with some peculiarity of the velocity distribution. Observations of velocity distribution, although very difficult near the boundary at which conditions are important for explanations, showed the existence of a thin layer of non-turbulent flow for so-called "smooth boundary"[5] and indicated, as implied by D'Arcy long ago,[8] that there was a universal velocity deficiency law within an inner somewhat indefinite zone[6,40,41] in the sense that, to suitably chosen nondimensional coordinates all velocity data plotted from their turning points as origin would indicate one curve there (See Fig. 3). A logarithmic curve is a good fit to the inner zone and to the wall zone at which one curve is not strictly a fit. Prandtl[40] has indicated conditions (not those of a circular pipe) under which a velocity distribution curve could be deduced to follow a logarithmic law over a range. A parabola is a good fit, according to Stanton,[5] over most of a pipe but does not attempt to fit the non-conforming wall zone. A hyperbolic function, according to the writer, fits a trifle better than a parabola. No doubt mathematicians could find comparably good fits. Now, with a formula fitted to a velocity curve a different way of looking at a flow formula becomes convenient. The flow formula may be regarded as the consequence of integrating

the velocity distribution (not the deficiency one) over the cross section to obtain Q, and dividing by the cross sectional area to obtain U. If the formula used to express approximately, the true velocity curve is a good practical fit over most of the pipe then the flow formula deduced from it by integration must be practically useful over the same ranges of variables. However, as the velocity formula remains speculative, the chances of deducing OB's dynamically correct flow formula from it seem remote. EFMI wrongly accepts the logarithmic fit to the velocity deficiency distribution over the whole pipe as absolute dynamical truth and uses it with various assumptions to deduce a logarithmic flow formula. This procedure he wrongly attributes to Prandtl despite the latter's warnings.[40]

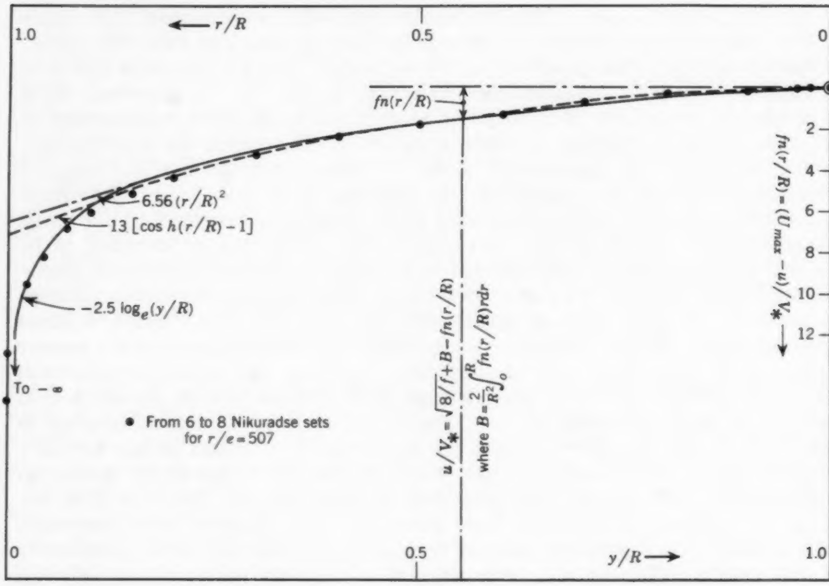

FIG. 3

With the preceding background we can now outline attacks along HI lines that should automatically answer specific parts of the comments.

First let us consider velocity distribution in a circular pipe under the given conditions and use dimensional analysis. Then we may write:

$$u = fn\left(y, R, U_{max}, e, \nu\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

and

$$\tau_o = fn\left(R, U_{max}, e, \rho, \nu\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

in which y is the distance of a point of interest from the boundary (assuming that the boundary can be replaced by a geometric surface), R is pipe radius (with the same proviso), u is the time-mean velocity at y, τ_o is the time-mean shear stress at the boundary. Given the mathematical ability we could always eliminate e from Eq. 23 by inserting its value from Eq. 24, and the

answer would be of the form:

$$u = \text{fn}\left(y, R, U_{max}, \tau_o, \rho, \nu\right) \quad \ldots\ldots\ldots\ldots \quad (25)$$

which can be reduced to:

$$\frac{u}{V_x} = \text{fn}\left(y/R, U_{max}/V_x, V_x\, y/\nu\right) \quad \ldots\ldots\ldots\ldots (26a)$$

Now, suppose the smooth-boundary case shown by the f diagram, in which e can be dropped from Eqs. 23 and 24. Then we could eliminate U_{max} from them and Eq. 26a would reduce to:

$$u/V_* = \text{fn}\left(y/R, V_*\, y/\nu\right) \text{ (Smooth)} \quad \ldots\ldots\ldots\ldots (26b)$$

It does not seem possible to believe both this equation and Fig. 102 of reference 15(a) quoted by Mr. Rajaratnam because that figure is fitted by a single line implying that y/R can be neglected. Also, if that line is really the ultimate truth, then we must believe that du/dy has a definite value at the centerline of a pipe, that is, there is a cusp in the velocity distribution. It is interesting to note that the line has an equation that gives a mixing length distribution in the inner 30% of the radius that is utterly different from that found by inserting measured values in the formula that defines mixing length.[43] The writer emphasizes that he is not seriously disputing practical goodness of fit of the line over the range of its data, but is disputing its dynamical exactness and, thence, the EFMI implication that the dynamically correct flow formula has been discovered beyond doubt, unlimited extrapolation is permissible, and the subject is forever closed.

Reverting now to Eq. 26a we note that it is consistent with the general experimental finding that, for any boundary, there is a universal velocity deficiency curve provided y is not too small. For it does not contradict the physical possibility of:

$$\left(U_{max} - u\right)/V_* = \text{fn}\left(y/R, V_* y/\nu\right) \quad \ldots\ldots\ldots\ldots \quad (27)$$

and f-diagram experience shows that viscosity can cease to affect certain problems appreciably when Reynold's number exceeds a certain amount. Further, arguing as we did to derive Eq. 26b from Eq. 26a, but eliminating ν instead of U_{max}, we would get

$$u/V_* = \text{fn}\left(y/R, U_{max}/V_*\right) \ldots\ldots\ldots\ldots \quad (28)$$

that would not contradict the possibility of nature arranging:

$$\left(U_{max} - u\right)/V_* = \text{fn}\ (y/R) \ldots\ldots\ldots\ldots \quad (29)$$

for this smooth boundary case. However, the function would be discontinuous because the existence of a laminar zone has been demonstrated experimentally.[5]

This seems to sum up all that is definitely known about velocity distributions. For practical reasons, or for analysis, various deductions may be made from various assumptions, but the results cannot be dynamically correct generally except by fortunate cancellation of errors. For example, the "thickness

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43 "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, p. 73, Fig. 49.

of the laminar sublayer . . . given by the equation . . . (11)" that Mr. Rajaratnam quotes on the authority of reference 15 is one of many quantitative definitions possible in terms one of many possible assumptions. It is derived by assuming that the laminar zone (reference 15, Fig. 102) is fitted by a straight line $\tau_o = \mu\, du/dy$, instead of a parabola as it should be for dynamical and functional correctness, and that the turbulent zone is fitted by the logarithmic formula that we have just demonstrated cannot be dynamically correct. The intersection of these two is found by trial and error solution of $V_* \delta/\nu = N = 2.5 \log N + 5.5$. Some proofs seem to make a kind of physical argument out of the constancy of $V_* \delta/\nu$, and thence claim special validity. Actually any single curve that fits the closely bunched points of Fig. 102 reference 15 will have a single intersection with any single curve that is supposed to fit the laminar film so will give a fixed $V_* \delta/\nu$ for the intersection. It should not be forgotten that the laminar film is of indefinite and fluctuating size that cannot be measured, so its "thickness" is merely a conventional measure. However, the convention may be based on a dynamically correct relation or not, so the definition might lead back to something useful. The writer chose to define δ in terms of an argument of Prandtl's adapted from a slightly different case;[6] allowing for the actually fluctuating motion in the laminar film the argument can be used almost word for word. The different definitions will not agree nor will results deduced from them.

Another class of not quite perfect deduction concerns flow formulas and suggests that there is not necessarily a mathematical objection to channels and pipes having the same functional type of flow formula—a point that interested Mr. Lacey. One starts from the deficiency law:

$$\left(U_{max} - u\right)/V_* = \text{fn}\,(r/R) \quad \ldots\ldots\ldots\ldots\ldots \quad (30)$$

and averages the terms over the cross section (by multiplying by $2\,\pi\, r \times dr$ integrating over the whole cross section, and dividing by $\pi\, R^2$). No matter what one assumes the average value of U_{max} is itself, and of u is U. The average value of the right side depends on the function and, of course, the experimentally determined values of the parameters in the function. If the function is taken as $A\,(r/R)^2$, in which $A = 6.56$ agrees with Stanton over 80% of R, the average is A/2, from the property of a paraboloid of revolution that the mean ordinate is half the maximum. If the function is the standard logarithmic, minus $2.50 \log_e y/R$, that attempts to fit the non-conforming wall zone, the average is 3.75 and is practically more accurate than $6.56/2 = 3.28$ that deliberately ignores the wall zone. The author has found that $13\left[\cosh(r/R) - 1\right]$ fits Nikuradse data very closely over 85% of R and gives an average of 3.435 (See Fig. 3). From the viewpoint of aiming at finding OB's dynamically perfect flow formula it is important to note that any function, fn(r/R), will lead to:

$$\left(U_{max} - U\right)/V_* = B = \frac{2}{R^2} \int_o^R \text{fn}\left(\frac{r}{R}\right) r\, dr \quad \ldots\ldots\ldots \quad (31)$$

in which B is a constant, and even if the whole process is repeated for a broad channel the only difference in Eq. 31 will be in the value of B (see Eq. 34). However, as was shown via Eq. 26a, the right hand side of Eq. 27 is not quite a function of r/R alone, so B should be slightly variable, and some doubt arises whether we can ever obtain a dynamically correct flow formula from B = constant. To preceed from Eq. 31 towards deducing flow formulas we use the definition $f = 2\ g\ dS/V^2$ and the dynamical fact that $\tau_o = \frac{1}{4}\ g\ dS$, so that

Eq. 31 converts to:

$$U_{max}/V_* = \sqrt{8/f} + B \quad \text{.............. (32a)}$$

or

$$U_{max}/U = 1 + B\sqrt{f/8} \quad \text{.............. (32b)}$$

leading to

$$u/V_* = \sqrt{8/f} + B - fn(r/R) \quad \text{............. (32c)}$$

Eq. 32a shows that no further progress towards deducing the form of f is possible without knowledge of the relation of U_{max}/V_* to the factors on which it depends; this knowledge is not contained in the velocity deficiency Eq. 27. The HI method of obtaining the knowledge would start by obtaining data and studying them. The writer would not like to assert, without extensive research into literature, that such data as were obtained preceded a decision to believe the logarithmic type of answer of engineering textbooks. For "rough boundary" one can quickly obtain U_{max}/V_* and e/r data of the Nikuradse's experiments on artificially roughened pipe from the diagrams in reference 41. If the reaction between the two variables is to be proved logarithmic the data are plotted on single log paper and will give an excellent straight line. If they are to be proved exponential the data are plotted on double log paper and will give an equally excellent straight line of slope 1/8. If the equation of the former line is inserted in Eq. 32a the standard logarithmic expression for f in terms of e/r will result. If the equation of the latter line is inserted it will not give the writers fourth-root Eq. 5 but would not be expected to give the latter's exact functional form even were it lucky enough to be the ultimate dynamically correct one. (An important lesson of these alternative plots is that a logarithmic function can agree with a suitable power one over large range of variable).

The relation of U_{max}/V_X to what causes it for smooth boundary does not seem to have been attempted directly. Instead, the sequence of indirect deduction seems to have been that explained neatly in reference 12, Sections 25 and 28. Briefly, Prandtl deduced velocity distribution along a plane wall in an infinite fluid with constant shear stress throughout and far enough from the wall for viscosity to be ineffective (reference 40, pp. 125-129). Having deduced it as logarithmic he proceeded, by an ingenious but not quite convincing argument, to extend it to fit the zone where viscosity mattered and came out with the expression:

$$k\, u/V_* = \log V_*\, y/\nu + C_i \quad \text{............. (33)}$$

He then pointed out that it would fit Nikuradse's experiments on smooth pipes "as an approximation" and that "for flow in pipes" there was actually a "deviation from theory." The writer does not know of his having recommended or supported, as dynamically exact, integration of this flow formula to give a mean value of U, and thence an expression for f. However, it is this integration with the constants fitting Nikuradse's smooth pipe f data, that gives the logarithmic smooth boundary line on the Moody diagram, and its formula is found associated in textbooks and articles with the names of Prandtl and von Karman in a way that seems to have caused engineering readers to believe that these hydraulicians deduced and recommended it. So, when Mr. Rajaratnam says "it is widely known that for values of the Reynolds number higher than 10^5 the flow resistance is better approximated by the Prandtl-Karman equation . . ." the writer feels that he has been misled by ESMI and the state-

ment should read "it is widely mis-stated by textbooks that . . . by the wrongly-named Prandtl-Karman equation." All that the Nikuradse smooth-pipe f values show is that even a so-called "smooth" pipe starts to become rough at high enough Reynold's numbers. If his "smooth" pipes had been replaced by even "smoother" ones his smooth line should have been expected to start turning up at higher Reynold's numbers still; if they had been a little less "smooth" then presumably he would have used them to obtain the constants in the logarithmic formula although they now pass off as extensions of the Colebrook-White ones and not "smooth." Problem 5 was framed deliberately for the interested reader so that he could decide for himself whether there was something dubious about the logarithmic smooth boundary formula at Reynolds Numbers higher than those used in the lab. Problem 4 was to convince him that, if the Moody Diagram is wrong, it may be alarmingly wrong for very large conduits.

A definite quantitative answer to the queries about canal distributions is prevented by inadequate data. As the writer's interest in flow formulas arose from the possible dynamical implications of Mr. Lacey's regime formulas, he has never been impelled to arrange observations of his own to determine the constant in pipe velocity deficiency formulas that would suit channels. His inquiries from laboratory workers on channels and his long field experience with canals support, very strongly, the view that the functional form of the velocity distribution down the centerline of a broad canal is that along the radius of a pipe. Narrowness probably does not change the functional form except by introducing an extra non-dimensional ratio such as b/d and altering the value of a parameter. Exact determination is difficult in laboratory flumes because they are narrow and the absolute depth is small. In the field current meters are useless near the free surface and the bed, and many beds are mobile. Moreover, even in lined canals the water is often turbid and laboratory observations show that turbidity can alter, radically, the coefficient 2.5 in the logarithmic deficiency formula—one result given privately by a well-known authority showed a change to 4.8 with a suspended load concentration of 15.8 grams per litre, but a very good agreement with 2.5 over 85% of the depth for clean water. Finally wind is a major disturbing element in the field. A mathematical approach to the problem seems to show a contradiction between the two common practices of assuming that the velocity deficiency expressions are the same, complete with constants, over the radius of a pipe and the depth of a broad canal and that, in use of the Moody diagram or the Manning formula, a channel can be replaced by a circle of the same hydraulic radius. If the velocity expressions are the same then the right hand of Eq. 31 becomes:

$$B = \frac{1}{R} \int_0^R \mathrm{fn}\left(\frac{r}{R}\right) dr \quad \ldots\ldots\ldots\ldots\ldots \quad (34)$$

that will not give the same value of B for the canal and the pipe. (Here r is depth from surface and R is overall depth). But if this is so then Eq. 32a shows that the velocity distributions are different for the two cases, in which it is supposed that V_x and f are the same. This raises the question whether the parameter in the velocity formula might differ between pipe and channel in such a way as to make B + fn (r/R) the same for both. None of the functions of this article seems adequate for the purpose, but a mathematician might discover one. However it should not be forgotten that B is not quite constant in fact. Physically the writer has not been troubled by the difference between

surface and center flow of canal and pipe. He has visualized mean surface flow in terms of the mathematical identity of the two cases (1) a vortex approaches a rigid plane wall and (2) equal and opposite vortices approach each other. But another way he has visualized the mean motion at the surface of a channel as if the mirror image were superimposed to turn it into a pipe. Perhaps the whole preceding discussion will explain why the writer regards the integration of speculative velocity distributions to obtain a flow formula as analogous to using the presumed behaviour of the constituents of the atom to obtain the simple laws of electromagnetism. The cautions procedure seems to be to use Eq. 32c to fix the constants of a practical velocity distribution in terms of f.

The commentators' interest in the justification for using channels to aid in testing for a universal flow formula obviously calls for a little explanation in amplification of the references. The writer's interest in dynamics and physics, his years of practical canal engineering, and his study of the success with which Mr. Lacey developed his regime theory along the logical lines of classical physics[44] have all helped him to think of the flow formula in terms of either the equating of resistance to driving force or the selection, by nature, of one particular flow for any one problem from among the infinite number of flows that one can imagine as kinematically possible. It is difficult, with an interest in physics, to believe that a formula giving the relationship between dynamical entities in simple observed circumstances will be complex, or that its functional form will be altered seriously without a radical change in the dynamical set-up. Complications may be added by geometric conditions, or by extraneous phenomena being measured with the ones of interest, but behind them is the simple relation. Fundamentally, in pipes of either kind of boundary, or in channels of mobile boundary, all carrying Newtonian fluids, the boundary "puts on the brake" and the flow responds by rolling-up into an eddying form that does not seem to be altered essentially by alteration in the nature of the brake. Why, then, should the functional form of the flow equation be different for the three cases or, for that matter, for channels with rigid boundaries? The answer seems to be that there should be no difference, but that the term that expresses the "braking action" will have different detailed expressions for the different cases. At one time the author tried to prove the point from somewhat advanced dynamical principles but lacked the necessary skill. However, the normal HI procedure that he likes to think of as "plotting formulas" instead of points, had to be followed. That is, a search had to be made for a thread running through the various competing formulas for smooth and rough rigid pipes and for mobile boundary canals (the last made available in physically suggestive form by Mr. Lacey's monumental work.[44] The thread discovered is the equation given in the paper under the heading "Universal flow formula," with its values of x. The "theory of flow," such as it is, amounts to believing, till something more definite turns up, that the odds against this functional form having no physical significance are very high. Accepting this belief the form merits a try in practical use and, far more important physically, ought to initiate a search for a dynamical theory that would prove or disprove it. It is not believed that the expressions for x are beyond improvement; in fact, for the regime case, it seems very likely that small b/d ratios will require an adjustment. Obviously

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44 "Flow in Alluvial Channels with Sandy Mobile Beds," by Gerald Lacey, Proceedings, Instit. Civ. Engrs., Vol. 9, February, 1958 pp. 145-164, plus discussion, Vol. 11, pp. 219-251, October, 1958.

many more data are required from the field to answer Problem 5 finally and thereby test the theory decisively over the practical range for smooth boundary. The data will have to be numerous, and from different observers, because accuracy of measurement will not be too good and the danger of accidental bias is high even in laboratory work.

Specific replies to remaining points are:

(1) The Prandtl proof is not valid for a canal. The outstanding reason is that it rests on the assumption of constant shear stress, but canal shear is proportional to depth from surface. Flow of wind along the earth's surface probably provides nearly ideal conditions not too close to the ground and not too far up.

(2) Referring to the Webster and Metcalf data, the variation of n does not seem sufficient, in terms of reasonable errors of observation and the uncertainty of where to measure d in a corrugated pipe, to pass judgement on the matter. Note that, for a known discharge, n varies as the 2.67 power of diameter.

(3) The reference to Morris' work is appreciated. The author should have quoted it when he made the qualification, in the "Introduction," that the Moody diagram applied for ". . . rigid boundary roughnesses that can be expressed in terms of a single roughness height." The oversight is regretted.

The recent references by Mr. Van Sickle merit study in original. The writer believes they emphasize his own view that deviation between the logarithmic and the exponential formulas is too small over most practical ranges to permit a decision to be made between them. For the rough boundary logarithmic formula the best present test, in terms of real data, seems to be as in "Inconsistence of Manning Formula and Moody Diagram," and for the smooth formula the best test is to study the few very large conduits, with Reynold's Number greater than about 5×10^6, in which conditions are a bit off smooth and there are enough data to show that the transition curve is consistent with Blasius (in that it could bend into the Blasius line) and inconsistent with the logarithmic curve in that it heads right across it. Thus Fig. 1 of reference 29 is for Reynold's Number well below 10^6 and indicates only that the aluminum pipe appears to be just starting to go into transition on its way towards roughness; some of the points are below the log line a little and the Figure has the logarithmic line incorrectly drawn in its upper portion. In reference 31, Fig. 3 shows four sets of observations with marked inconsistencies that puzzled the commentator, Mr. Campbell, and caused the author, Mr. Burke, to admit that he believed conditions were not of uniform flow. Mr. Campbell noted that the one set of the four that gave the approximate trend of the logarithmic smooth boundary line fell below it. A very interesting feature of the reference is the plot of observed velocity distribution in its Fig. 7. This plot deviates from the so-called Prandtl one (that was not for pipes) in the inner 90% of the pipe in the manner of the hyperbolic cosine curve of Fig. 3 (herein), but to a greater extent.

Finally, references 35 and 36 are very informative but do not seem to add significantly, for purposes of the present discussion, to the discussion of the paper of Hickox, Peturka, and Elder. They include Raam's method of estimating f for rock tunnels—a method that the author likes because it defines a practical method of measuring a parameter obviously related to roughness and correlates it with f. This procedure ought to make the f-n diagram, or any other, useable outside the range where roughness height becomes large compared with diameter and is on a par, for practical utility, with using experience, rather than formal curves, in the transition zone.

UNIFORM WATER CONVEYANCE CHANNELS IN ALLUVIAL MATERIAL[a]

Discussion by Marcel Bitoun

MARCEL BITOUN,[10] M. ASCE.—The writer was interested in this paper from the standpoint of the engineer who is confronted with the design of large irrigation canals. The scope of the material covered is the widest yet published, and their review of the data collected by others and by themselves is a most valuable contribution to the study of the behavior of man-made alluvial channels.

Careful reading of the paper and examination of the basic data reveal that most of the canals carrying flows smaller than 1000 cfs follow the various correlations studied relatively well. However, as was pointed out, the larger

TABLE 8.—COMPUTED MANNING COEFFICIENTS

Canal Number (1)	Discharge, cfs (2)	n (3)
2	9,005	0.028
3	4,463	0.028
7	1,081	0.024
8	2,824	0.027
9	4,420	0.028
13	2,102	0.018
27	1,187	0.021
31	1,408	0.024
38	5,677	0.027
41	5,810	0.025

canals, conveying flows in excess of 1000 cfs, seem to depart markedly from the general trends indicated by some of the correlations. In particular, Fig. 8 shows a limitation of the mean velocity to approximately 3 fps in the Punjab and Sind canals when R^2S becomes larger than 0.005.

This can be expressed differently: if the Manning roughness factors n are computed by application of the Manning formula to the Punjab canals, one obtains values substantially higher for the larger canals than for the smaller canals. For comparison purposes, an average value of n for the smaller canals is 0.0215. See Table 8.

The Blench-King formula, Eqs. 15 and 29, is also inaccurate. Table 9 shows the results of the application of the Blench-King slope formula and the Lacy formula to the large canals.

[a] May 1960, by D. B. Simons and M. L. Albertson (Proc. Paper 2484).

[10] Harza Engrg. Co., Chicago, Ill.

It is apparent that the application of these formulas lead to slopes substantially flatter than the actual slopes.

Although the authors indicate that a reasonable estimate of the design slope can be obtained by using the correlation of V and R^2S or by using the Blench-King regime slope formula, this accuracy does not appear to be sufficient for design purposes, as the determination of the slope of irrigation canals is one of the most critical items.

A reliable formula for flow on movable alluvial materials should give consideration to the grain size of the bed material and to the bed configuration. Such a formula was proposed by H. K. Liu, M. ASCE and S. Y. Hwang, A.M. ASCE (69). The validity of its form was verified on the Punjab canals, and computed slopes for the large canals are shown in Table 10.

The coefficients C_b listed in Table 10 are slightly different from the coefficients C_a recommended by Liu and Hwang. They were derived by the writer from an analysis of the Punjab data. The necessity of making minor

TABLE 9.—COMPARISON OF REGIME SLOPE FORMULAE

Canal No.	Measured Slope (per thousand)	Computed Slope	
		Blench-King	Lacey
(1)	(2)	(3)	(4)
2	0.19	0.13	0.09
3	0.20	0.13	0.10
7	0.21	0.19	0.18
8	0.21	0.15	0.12
9	0.20	0.14	0.11
13	0.18	0.15	0.12
27	0.13	0.17	0.14
31	0.17	0.16	0.13
38	0.13	0.11	0.06
41	0.12	0.11	0.06

adjustments to C_a for application of the formula to different areas is attributed by the writer to the fact that the mean grain size is not sufficient to completely define a bed material.

The accuracy of the formula

$$V = C_b\, R^x\, S^y \qquad (70)$$

is definitely better than that of the regime slope formulas recommended by the authors. It is within acceptable limits for design purposes and it is suggested that its use be considered.

Another anomaly is the fact that the Punjab canals, reputed to be stable, are characterized by widely different tractive forces on the bed and largely different width-depths ratios. According to Lindley, Lacey, and Blench, for canals constructed in similar soils, the relationship between the width-depth ratio and the velocity is

$$\frac{W}{D} = K\, V \qquad (71)$$

Analysis of the Punjab data, whose soils are described as relatively uniform, does not confirm this relationship.

The correlation of W/D and V is shown in Fig. 17(a). The width used in this correlation is as defined by Blench: W = A/D.

It is apparent from Fig. 17(a) that the various canals are characterized by widely different values of K. For example, in comparing large canals No. 2, 3, 7, 8, and 9, the bed material mean diameter varies only from 0.41 mm to 0.43 mm, and V varies from 2.54 to 3.25 fps, whereas K varies from 4.75 to 7.75. For the ensemble of the Punjab canals, K is comprised within an even wider range: 0.70 to 7.75.

However a comparison of groups of canals operating under similar conditions of velocity reveals the following:

1. For the same mean diameter of bed material, the tractive force on the bed increases up to a certain value as the width-depth ratio decreases.
2. The maximum value of the tractive force exhibited by the canals and the mean diameter of the bed material both increase as the width-depth ratio decreases.

TABLE 10.—SLOPES COMPUTED BY THE LIU-HWANG FORMULA[a]

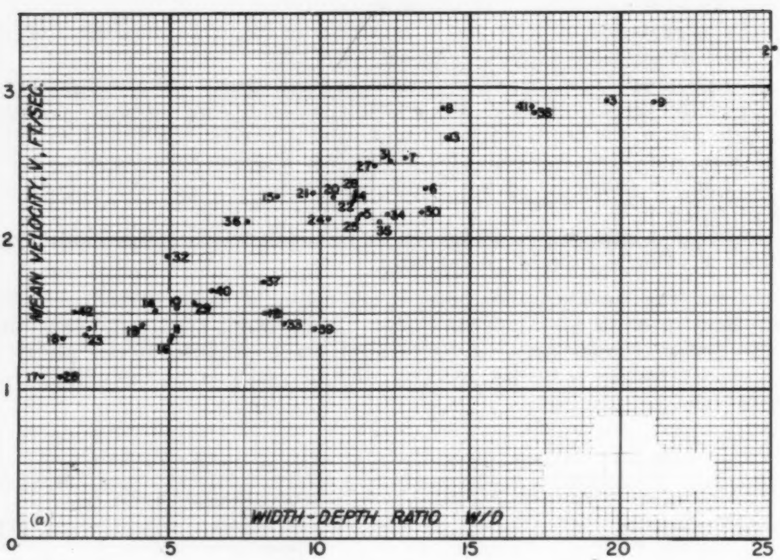
Canal No. (1)	Bed Material (d_{50}, mm) (2)	C_b (3)	x (4)	y (5)	Computed Slope (per thousand) (6)	Measured Slope (per thousand) (7)	Blench-King[b] (8)	Lacey[c] (9)
2	0.42	18.3	0.50	0.33	0.18	0.19	0.13	0.09
3	0.42	18.3	0.50	0.33	0.18	0.20	0.13	0.10
7	0.41	18.3	0.49	0.325	0.22	0.21	0.19	0.18
8	0.43	18.5	0.50	0.33	0.20	0.21	0.15	0.12
9	0.41	18.3	0.49	0.325	0.17	0.20	0.14	0.11
13	0.34	18.1	0.47	0.32	0.17	0.18	0.15	0.12
27	0.31	18.0	0.46	0.315	0.16	0.13	0.17	0.14
31	0.35	18.1	0.47	0.32	0.17	0.17	0.16	0.13
38	0.30	18.0	0.46	0.315	0.11	0.13	0.11	0.06
41	0.30	18.0	0.46	0.315	0.11	0.12	0.11	0.06

[a] $V = C_b R^x S^y$
[b] Slopes computed by the Blench-King formula as shown on Table 9.
[c] Slopes computed by the Lacey formula as shown on Table 9.

To illustrate this, Table 11 shows the values of τ, d_{50} and W/D for the group of canals operating at velocities of approximately 1.4 fps.

It is suggested that some of the canals could have been constructed with a lower value of W/D without danger of bed scouring. For example, canal No. 33, in which the tractive force is 0.026 psf could have been operated with a tractive force as high as that of canal No. 14 (0.044 psf) without being eroded.

Thus, instead of searching for relationships common to all the canals under investigation, envelope curves corresponding to the limit of stability should be defined. Such curves have been drawn on Fig. 17(b). They represent the minimum width-depth ratios that characterize the stable Punjab canals and should correspond to tractive forces on the bed close to the limit of stability



(b)

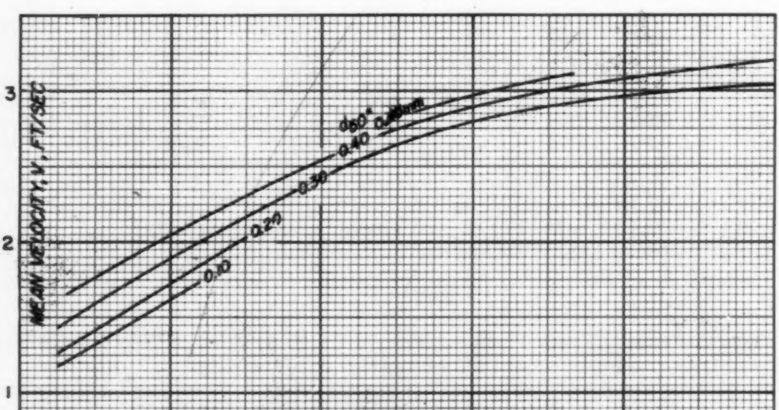


FIG. 17.—PUNJAB CANALS

compatible with a mean grain size of bed material. These curves are presented here only in order to illustrate the concept of limiting conditions. Further investigations are necessary to determine accurate numerical values.

The writer grouped the forty-two Punjab canals according to their eleven median grain sizes, then selected the highest tractive force value for each group and tabulated these values opposite their respective grain sizes as follows:

d_{50} (mm)	max τ, in psf
0.43	0.110
0.42	0.111
0.41	0.106
0.34	0.084
0.30	0.088
0.29	0.052
0.28	0.047
0.26	0.054
0.19	0.048
0.17	0.049
0.15	0.050

TABLE 11[a]

Canal Number (1)	W/D (2)	τ in psf (3)	d_{50}, in min (4)	Group (5)
18	1.5	0.048	0.19	1
42	1.8	0.032	0.18	1
23	2.2	0.043	0.20	1
1	2.4	0.054	0.26	3
19	4.2	0.041	0.19	1
14	4.5	0.044	0.22	2
16	5.0	0.042	0.22	2
11	5.1	0.036	0.23	2
10	5.3	0.043	0.26	3
29	5.9	0.043	0.26	3
12	8.2	0.036	0.28	3
33	8.8	0.026	0.23	2
39	9.8	0.023	0.20	1

[a] Group 1 corresponds to d_{50} = 0.18 - 0.20 mm
Group 2 corresponds to d_{50} = 0.22 - 0.23 mm
Group 3 corresponds to d_{50} = 0.26 - 0.28 mm

These values of τ can be compared to the ones recommended by E. W. Lane for stable canals. Fair agreement is shown by Fig. 18.

It was previously mentioned that, for similar mean velocities, the bed material in the stable Punjab canals becomes coarser as the width-depth ratio decreases. This is put in evidence by the family of envelope curves shown on Fig. 17(b). Considerations of fluvial dynamics indicate that when balanced regime is established, the material that forms the bed of an alluvial channel is characterized by a coarser mean diameter than the original soil in which the

channel was excavated. During the transition period that precedes establishment of regime conditions, the material in place is subjected to a process of sorting or segregation. The fines contained in the bed material are put in suspension by turbulence and carried away as wash load. If the material in place contains a sufficient proportion of grains with a size such that they cannot be removed, an armor of a certain thickness develops on the bed and prevents further segregation. The median size of the armor increases with the

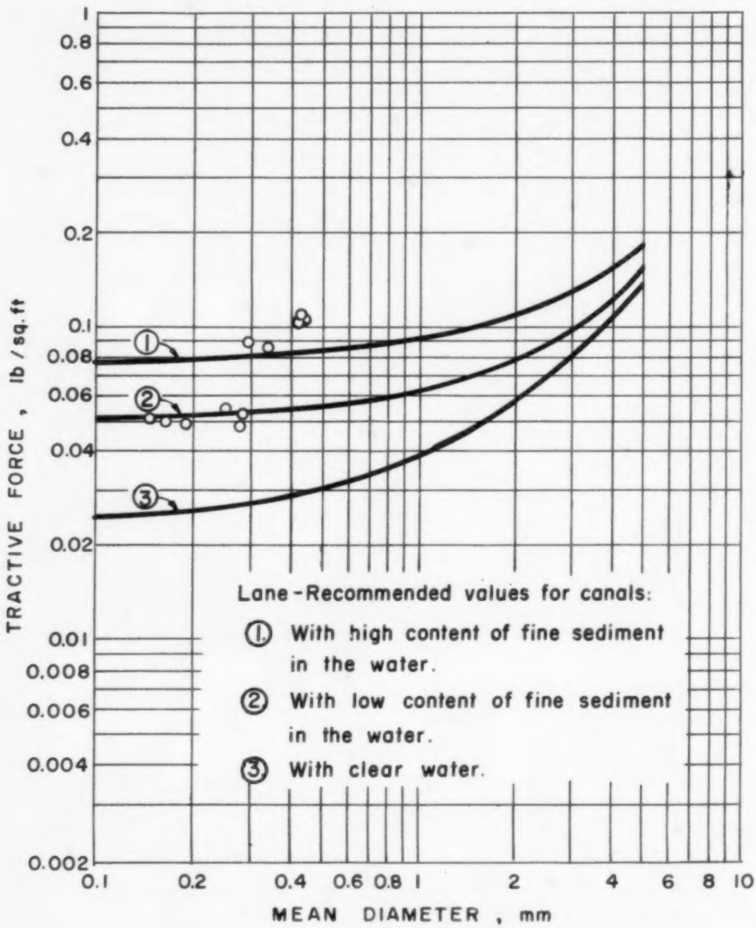


FIG. 18.—LIMITING TRACTIVE FORCES

forces acting on the bed. However, although this bed is not being eroded in a stable alluvial canal, it is moving as bed load. Stability requires that the bed load movement of the canal bed be replaced by a supply of solids into the canal at its headworks, equal in size and in quantity.

In the case of the Punjab canals, it could be that if some of the wider canals (with relation to depth) had been constructed somewhat narrower, their bed material would have been segregated to a larger size without causing further instability. They did not silt probably because the solid charge was not excessive.

It would thus be possible to predict the extent to which segregation will occur in a new canal on the basis of the tractive forces imposed, the gradation of the material in place, and the solid charge permitted to enter the canal at the headworks (or imposed by the parent river when sand control devices are not present). The design can be conducted so as to allow segregation affecting the bed material up to a large grain size, if such grains are available in a sufficient amount in the material originally in place. This would lead to lower width-depth ratios. However, the width is limited in practice by the fact that erosion of the banks could then occur. Stability of the banks could be determined by relating the tractive force on the banks to the friction angle of the material in place and the slope of the cut (or fill).

However, stability cannot be insured unless the bed load movement is determined for the chosen size of canal and segregated bed material size, and unless the supply of sediments into the canal at the headworks is maintained equal in size and in quantity to the material migrating downstream as bed load.

When discharge and sediment concentration in the parent river is variable throughout the year, stability of the canal must take this into account. The variation of sediment concentrations should be correlated to the variation of discharge in the canal imposed by irrigation requirements. An integration of the aggradation and degradation should provide a yearly cycle resulting in overall stability.

In that respect, it would have been interesting to find among the data presented detailed information on seasonal variations of flow in the canals investigated and on the sediment allowed to enter the canals.

An approach to the regime theory should not be based on a statistical analysis producing averages. The recognition of limiting conditions leads to results equivalent to those obtained from an approach based on the tractive force theory. These approaches, however, should be supplemented by due consideration given to solid transportation as one of the determining factors of stability. Addition of a bed load formula to the basic equations of stability is necessary and should enable a complete determination of the regime of equilibrium. Yearly average stability implies an integration of allowable aggradation and degradation with respect to fluctuating sediment supply and water requirements.

ADDITIONAL BIBLIOGRAPHY

69. "Discharge Formula for Straight Alluvial Channels," by H. K. Liu and S. Y. Hwang, Proceedings, ASCE, November, 1959.

MONIR M. KANSOH.[11]—This study of existing stable canals in the United States is actually a continuation of the regime studies that were initiated in

[11] Prof. of Hydr. and Water Power Engrg., Alexandria Univ., U.A.R.

the Punjab by Kennedy and that yielded the empirical equation:

$$V_o = 0.84\ D^{0.64} \qquad (72a)$$

using foot units, or

$$V_o = 0.546\ D^{0.64} \qquad (72b)$$

using metric units.

The diameter of the suspended material in the River Nile in Egypt much less than 0.20 mm with a concentration varying from about 3500 ppm down to about 200 ppm with an average of about 1600 ppm, nearly similar to the conditions in the Sind in which the "mean size of bed material is within the limits 0.0346 mm to 0.1642 mm" and "the suspended sediment load ranges from 3590 ppm to 156 ppm." This is to be compared with the conditions in the Punjab in which "the average diameter of the bed material is approximately 0.43 mm" and the mean silt intensity is, like that in the canals investigated by Simons and Bender, about 328 ppm. According to the authors, the tractive force theory may not be applied in Egypt where water is not clear and where the bed and sides of the canals are, in most cases, composed of fine cohesive materials. Design of stable canals should, then, depend on regime formulas.

In Egyptian canals, because the silt is finer than that in the Punjab, the critical velocity is sometimes taken as $\frac{2}{3}$ of the value given by Kennedy:

$$V_o = 0.56\ D^{0.64} \qquad (73a)$$

using foot units, or

$$V_o = 0.36\ D^{0.64} \qquad (73b)$$

using metric units.

Observations on thirteen canals in different parts of Egypt by K. Ghaleb produced the empirical equation

$$V_o = 0.284\ D^{0.727} \qquad (74)$$

using metric units.

These three values are rather excessive and it was recommended, especially for canals in Lower Egypt, that the formula

$$V_o = p\ D^m \qquad (75)$$

be used in which $m = \frac{2}{3}$, whereas p varies between 0.26 (for fine stilt) to 0.30 (for coarse silt), using metric units.

The form of the canal cross section is supposed to be predetermined. If it is, as is usually the case, a trapezoidal cross section with a bed width B, a depth D and with sides sloping at an angle θ with the horizontal, then

$$A = (B + D \cot \theta)\ D = Q/V = Q/\left(p\ D^m\right) \qquad (76a)$$

$$B = (A/D) - D \cot \theta = Q/\left(p\ D^{m+1}\right) - D \cot \theta \qquad (76b)$$

$$P = Q/\left(p\ D^{m+1}\right) + D\,(2 \operatorname{cosec} \theta - \cot \theta) \qquad (76c)$$

and

$$R = \left(\frac{Q}{p \cdot D^m}\right) \Bigg/ \left[\left(\frac{Q}{p \cdot D^{m+1}}\right) + D\ (2 \operatorname{cosec} \theta - \cot \theta\right] \qquad (76d)$$

To fix the dimensions of a stable canal in order to convey a discharge Q, the critical velocity as given by the empirical regime formula is substituted

into the Chezy equation with the aid of one of the formulas given by Bazin, Manning, Kutter, and others. The Manning formula is often recommended because, in addition to its simplicity, it gives satisfactory results.

Then

$$V_o = p\,D^m = \frac{1.486}{n}\,R^{2/3}\,S^{1/2} \qquad (77a)$$

(foot units) or

$$V_o = \frac{1}{n}\,R^{2/3}\,S^{1/2} \qquad (77b)$$

(metric units).

The depth D can be computed after putting R in terms of D as previously shown.

The bed width can be directly computed because

$$B = \frac{Q}{\left(p\,D^{m+1}\right)} - D\cot\theta \qquad (78)$$

The computation is much simplified and even avoided by using diagrams, nomograms, or tables.

The relation between B and D, or between W and D might as well be obtained from empirical regime equations like Eq. 4, or Eqs. 13 and 14.

After a careful examination of a large number of recognized canals in Egypt, Molseworth and Yennidunia recommended the following relations between B and D:

$$D = 0.10\left(\frac{S}{2} + 4\right) B^{1/2} \qquad (79a)$$

and

$$D = 0.00154\,(S + 8)^2\,B \qquad (79b)$$

for D above 1.62 m and below 1.62 m respectively, S being the water slope in the canal in centimeters per kilometer.

They also recommended the ratios

$$D = C_1\,B \qquad (80a)$$

when B is less than 2.00 m and

$$D = C_2\,B^{1/3} \qquad (80b)$$

when B is greater than 2.00 m, in which $C_1 = 1.00$ and $C_2 = 1.75$ for deep drains while $C_1 = 0.90$ and $C_2 = 1.45$ for shallow drains.

These relations are represented in Fig. 19, that gives the desirable proportions of stable trapezoidal canals and drains with 1:1 side slopes. The diagram consists of two groups of curves:

1. The dotted curves that correspond to the value

$$K = A\,R^{2/3} = Q\left(\frac{1}{n}\,S^{1/2}\right) \qquad (81)$$

using metric units.

2. The ratio-curves corresponding to canals of different slopes, deep drains, and shallow drains.

The point of intersection of the appropriate K-curve and the proper ratio-curve determines the desirable depth and bed width of the canal.

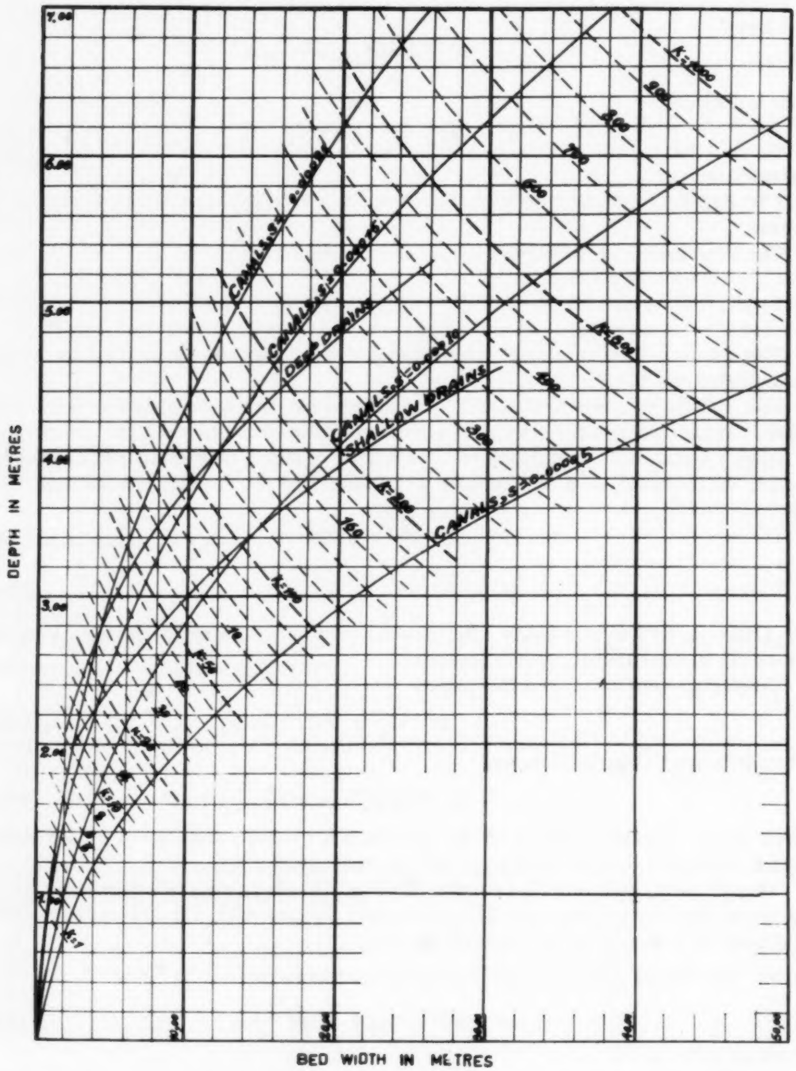


FIG. 19.—DESIGN OF STABLE TRAPEZOIDAL CANALS AND DRAINS WITH 1 ON 1 SIDE SLOPES

Interpolation is sometimes necessary when the computed value of $Q/\left(\frac{1}{n} S^{1/2}\right)$ lies between two K-curves, or when the slope lies between two of the slopes indicated by the ratio-curves.

The same relations of B and D, for canals, are the basis of a standard diagram used in the Egyptian Ministry of Public Works for the design and maintenance of stable trapezoidal canals with 1-on-1 side slopes when the solids in suspension do not exceed 2000 ppm. This diagram, Fig. 20, is suitable for one class of canals whose Manning roughness coefficient n = 0.025, while Fig. 19, suits any value of n.

To use the standard diagram, Fig. 20, a vertical straight line indicating the discharge is extended to intersect the right hand inclined straight line indicating the chosen or selected slope. From the point of intersection, a horizontal straight line is extended to the left. This horizontal straight line and the left hand inclined straight line corresponding to the same selected slope intersect at a point that directly indicates the desirable depth and bed width of the canal.

The procedure may sometimes be altered by fixing the depth and trying to find slope and bed width corresponding to the given or required discharge. It may be the bed width that is first known while the depth and the slope are to be determined.

Hesitation usually accompanies the selection of the suitable slope indicated by one pair out of the two groups of inclined straight lines. To reduce this hesitation, H. El Defrawy suggested a particular selected slope for each range of discharge, depth and bed width. These slopes are shown in thick lines in Fig. 20.

A. M. El Banna proposed a diagram, Fig. 21, that is a combination of the standard diagram, Fig. 20, and an old nomogram known as Wallace diagram.

For using Fig. 21, it is sufficient to extend the straight line in both directions joining the value of the discharge Q with the equal number on the scale line. The straight line will directly give the desirable depth, bed width, and slope. In choosing the number on the scale-line, a little adjustment is sometimes necessary so that the bed width may be a round number.

Adopting the dimensions given by this diagram, permits a velocity of flow claimed to be a critical one. This velocity is said to have the value

$$V_o = p_1 + p_2 D^m \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (82)$$

in which

$$p_1 = p_2 = 0.15 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (83)$$

m = 1.00 and D is the depth of flow when the canal conveys the maximum demand.

The authors state that Q could be correlated with D but, for design purposes, R is a more meaningful measure. This may explain why Q is given in terms of P and R in Figs. 3 and 5. In fact, neither P, R nor the top width is of a practical value in fixing the dimensions or proportions of a canal cross section as is the ratio B/D or W/D, and so on.

The many diagrams given in the paper are, in the writer's opinion, not more useful than any one of Figs. 19, 20 or 21. The former diagrams could preferably have been amalgamated into a single diagram, nomogram or abac, to give a direct solution for the problem of fixing the slope, dimensions and/or proportions of stable canals under different conditions.

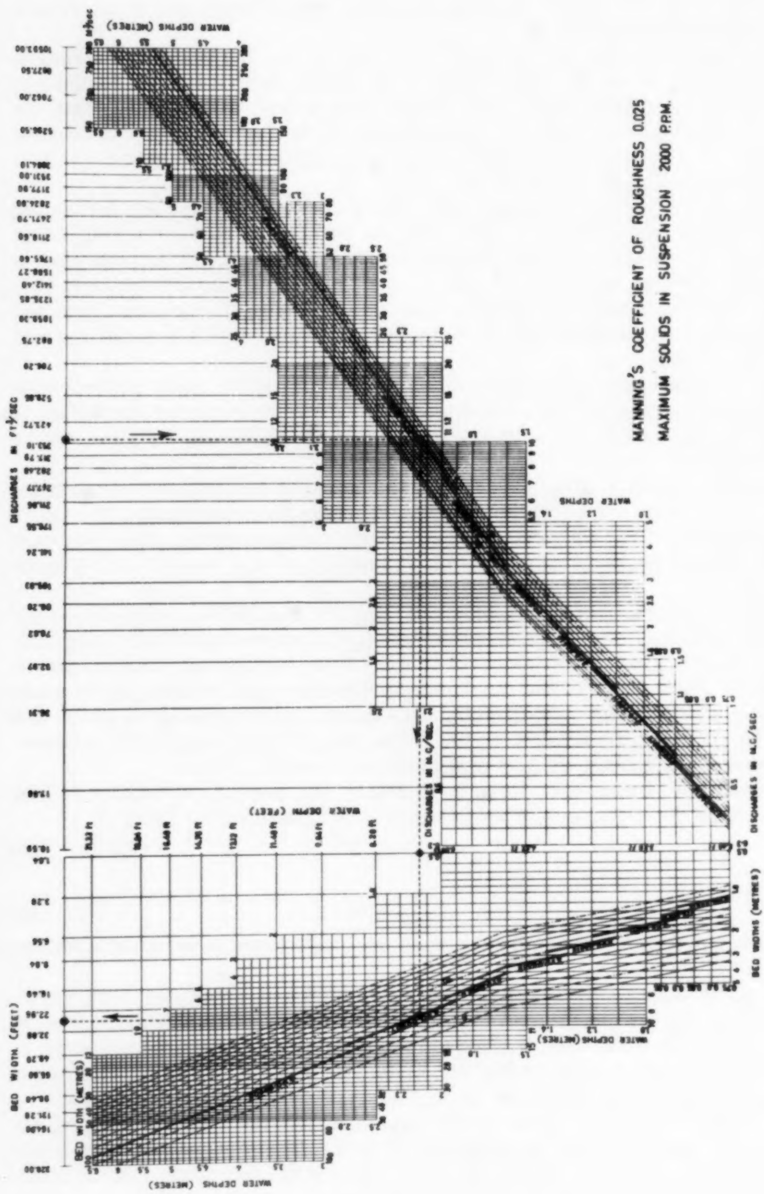


FIG. 20.—STANDARD TYPES FOR EGYPTIAN CANALS OF TRAPEZOIDAL SECTION WITH 1 ON 1 SIDE SLOPES

Nevertheless, the information given offers a direct relation or proportion of the average width and the depth of stable canals. The width depth ratio can be obtained as follows:

From Fig. 2 the perimeter P nearly equals 1.12 W. Noting Fig. 3 and Eqs. 9 and 19

$$W = \frac{2.668}{1.12} Q^{0.50} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (84)$$

approximately.

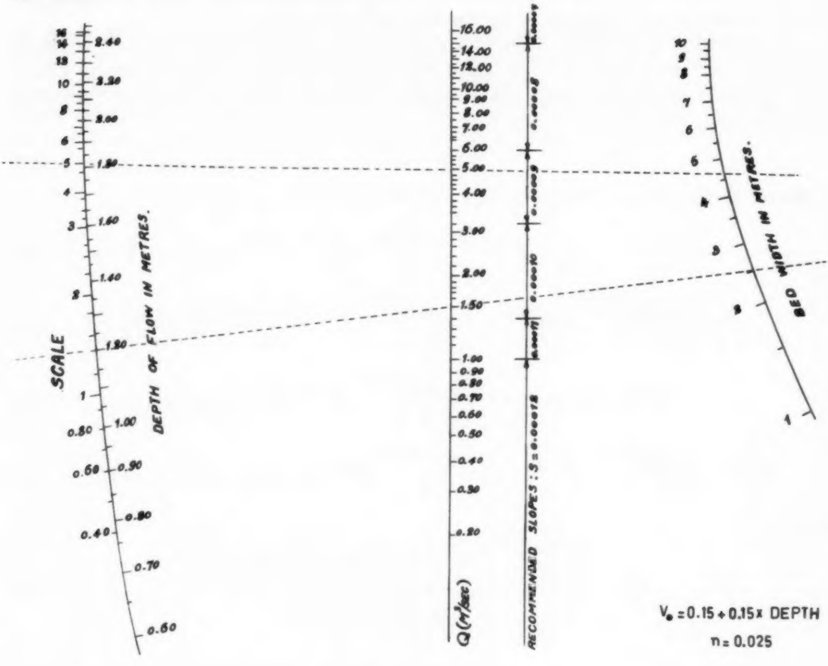


FIG. 21.—PROPOSED DIAGRAM FOR THE DESIGN OF TRAPEZOIDAL NON SILTING CANALS IN EGYPT WITH 1 ON 1 SIDE SLOPES

Similarly, taking the values of the hydraulic radius R from Fig. 1 (roughly 0.81 D) and from Eq. 12, from Fig. 5, and Eq. 21, for sand bed and natural berm, and from Eq. 22, for coarse non-cohesive materials, that is

$$D = \left(\frac{0.43}{0.81} \text{ or } \frac{0.247}{0.81}\right) Q^{0.361} \quad \ldots\ldots\ldots\ldots \quad (85)$$

Hence, the required ratio may be taken as

$$W/D = 4.5\ Q^{0.139} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (86a)$$

for sand bed and natural berm, or

$$W/D = 7.8\ Q^{0.139} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (86b)$$

for coarse non-cohesive material.

The last kind of material was mentioned, with this description, many times in the paper. The writer wonders if there exists a coarse cohesive material.

One of the important points is the limitation of the validity and applicability of regime equations that, as was stated, are valid only within the range of the observed data or are only valid for the limited range of conditions on which they are based. Two of these conditions were mentioned, in the same page, whereas one, a very important one with regard to the range of applicable slopes, was ignored. It is true that the range of slopes was mentioned as varying from 0.000058 to 0.000388, but this information is most confusing and most misleading as explained by the following example.

Considering a uniform flow in a stable trapezoidal canal of a bed width B, depth D and sides sloping at an angle θ to the horizontal, and combining Chézy and Manning formulas with the Kennedy equation that, in order to facilitate computation, may be put in the form

$$V_o = p\, D^{2/3} \qquad (87)$$

instead of the original form of Eqs. 72. Then

$$V_o = p\, D^{2/3} = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (88)$$

$$D = \left(\frac{1.486}{p\, n}\right)^{3/2} R\, S^{3/4} = \left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4} \left(\frac{(B + D \cot\theta)\, D}{(B + 2\, D \operatorname{cosec}\theta)}\right) \qquad (89)$$

$$B + 2\, D \operatorname{cosec}\theta = \left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4} (B + D \cot\theta) \qquad (90)$$

and

$$B\left[1 - \left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4}\right] = \left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4}\, D \cot\theta - (2 \operatorname{cosec}\theta)\, D \qquad (91)$$

Hence

$$\frac{B}{D} = \frac{\left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4} (\cot\theta) - 2(\operatorname{cosec}\theta)}{1 - \left(\frac{1.486}{p\, n}\right)^{3/2} S^{3/4}} \qquad (92)$$

If n is given an average value 0.025 and if θ is considered to be 45° and, supposing that the canal conveys water carrying fine silt such that p in Kennedy equation can be assumed less than $\frac{0.84}{2}$ or, say, 0.396 then

$$\frac{B}{D} = \frac{\left(\frac{1.486}{0.396 \times 0.025}\right)^{3/2} S^{3/4} (1.00) - 2 \times (1.414)}{1 - \left(\frac{1.486}{0.396 \times 0.025}\right)^{3/2} S^{3/4}} \qquad (93a)$$

$$\frac{B}{D} = \frac{1500\, S^{3/4} - 2.828}{1 - 1500\, S^{3/4}} = \frac{S^{3/4} - 0.00188}{0.000665 - S^{3/4}} \qquad (93b)$$

or

$$\frac{B}{D} = \frac{S^{3/4} - (0.000232)^{3/4}}{(0.000058)^{3/4} - S^{3/4}} \qquad (93c)$$

Real values of B and D necessitate that the ratio $\frac{B}{D}$ should be positive that means that in the present particular case of water carrying fine silt flowing uniformly in a canal of n = 0.025 and $\theta = 45°$, the slope S should vary from 0.000058 to 0.000232 only, whereas the slope is stated, in page 66, to vary from 0.000058 to 0.000389 without limitation or reference to any conditions.

Even in the comparatively narrow range of slope between 0.000058 and 0.000232, the ratio $\frac{B}{D}$ drops quickly from infinity to zero. Reasonable values of $\frac{B}{D}$ may then be expected only within the range of slopes from about 0.00010 to about 0.00020, for example, under the described circumstances.

In addition to the care necessary to design stable canals, measures must be taken to reduce silt troubles, for example, by feeding the canals and branches from the outer side of curved reaches of rivers and main canals, as well as providing the canals with proper intakes comprising sills, sand traps and similar precautions.

DRAG FORCES IN VELOCITY GRADIENT FLOW[a]

Discussion by Donald Van Sickle

DONALD VAN SICKLE,[6] A.M. ASCE.—The writer was associated with the authors at the planning stage of the investigation and designed and tested the force-measuring device that is described briefly in the paper under Mr. Moore's direction, for a previous investigation.[7] One of the primary aims in the development of this device was to make it adaptable for use in subsequent investigations, and it is gratifying to the writer that apparently this aim was achieved.

The authors very carefully point out that this was only an exploratory investigation, presumably because they felt that the limited amount of data did not permit more than general conclusions to be reached. There are, however, a number of points that could perhaps be given additional attention.

Eq. 4, in which, presumably, p is pressure intensity in pounds per square inch and r the cylinder radius in inches, gives the local force, f, in pounds per inch of height. For use in Eq. 3 however, f must be in pounds to give a dimensionless c_D. It would seem likely that the authors actually multiplied the "f" obtained from Eq. 4 by some unit of height, possibly the orifice spacing, to determine the local force to be used in Eq. 3. Could the authors clarify this point?

To the writer's knowledge, the variable shutter gate used for achieving the desired velocity gradients has not been reported previously in the literature. Could the authors provide more details on the construction and operation of the gate and the range of effects achieved? How does it affect transverse velocity profiles? Is it possible to achieve a negative velocity gradient, that is with the maximum velocity at the channel bottom? Where were the velocity profiles of Fig. 4 taken with respect to the gate and the measuring cylinder? For how great a distance downstream did the gradient persist before boundary layer formation produced a more normal velocity distribution?

The statement is made that "the measured total drag included, of course, the wave drag at the free surface. An approximate evaluation of the wave drag was made, based on the profile of the wave surface around the cylinder." Could the authors give more details concerning the procedure for evaluating wave drag and, in addition, some quantitative indication of how "satisfactory" the agreement was between measured and computed total drag? The total c_d values for the various runs would be of interest.

In the description of equipment the authors mention 24 piezometers in the measuring cylinder. Fig. 3 shows curves for only 10 of these, with numbers

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[a] July, 1960, by Frank D. Masch and Walter L. Moore (Proc. Paper 2546).

[6] Hydr. Engr., Turner and Collie Cons. Engrs., Inc., Houston, Tex.

[7] "An Experimental Investigation of the Drag Characteristics of a Model H-Section Bridge Pier," by Donald VanSickle, Thesis presented to the Univ. of Texas at Austin, Tex., in 1958, in partial fulfillment for degree of Master of Science.

1, 4, 7, and 14 through 24 being omitted. Similarly, on Fig. 5, the local drag coefficient is shown for eight points in one graph, ten points in another, and thirteen points in the others. Because the depth was the same for all cases, presumably the same number of points would have been available for all. Could the authors explain the missing piezometers?

In Fig. 4, the upper two curves indicate that velocity measurements were taken at the water surface, because the depth was stated to be 0.8 ft. Were readings actually taken at the surface, or was the depth for these curves greater than 0.8 ft?

The label for ordinates in Figs. 4 and 5 is more correctly given as "Distance from the Bottom, Feet" as described in the text.

Based on the stated flow depth of 0.8 ft and the average velocity of approximately 2 fps, the Froude Number for the authors' tests would be on the order of 0.4. In the writer's experience with the equipment used, Froude numbers of this magnitude and higher caused some oscillation of the cylinder, even with the dash pot arrangement. Did the authors find that these oscillations of the cylinder caused similar pressure oscillations on the manometer bank? If so, was any difficulty encountered in taking readings on 13 (or 24) manometers simultaneously?

In Fig. 1, the arrows indicate that on the upstream side of the pier, at the channel bottom, the induced flow is in the upstream direction. Was any attempt made to inject dye from the bottom piezometer to check this?

Had the authors considered the possibility that the surface wave could be at least partly responsible for the induced flows? With a standing wave at the upstream stagnation point, and a "standing depression" at the downstream stagnation point, the differential head between the stagnation points and the surrounding fluid could induce flows similar to those observed.

In Fig. 3, those portions of the curves lying between 90° and 270° are shown as being positive. Because the cosine in this region is negative, the pressures must also be negative to produce a positive product. Are these pressures negative with respect to surrounding fluid pressure or some other datum and how was the "datum pressure" measured? Because the cosine between 35° and 90°, and between 270° and 325° is positive, negative pressures must obtain between these points to give the negative product shown. Do the authors consider the 35° and 325° points (approximately) the points of separation? Do the authors consider it unusual that the absolute magnitude of the pressure at the downstream stagnation points for curves 10 through 13 is at least twice that at the upstream stagnation points?

The authors use wave forces on piers and wind forces on stacks as examples of engineering problems in which velocity gradients are of significance. It should be pointed out that in both types of problems pressure fluctuations and flow accelerations are also present. While the authors are evidently aware of this, they seem to give the impression that the velocity gradient is more important than acceleration of the flow. Do they feel that their results could, in general, be said to apply to flow with acceleration even though the experiments were performed in a steady, non-uniform flow?

The writer certainly does not wish to detract in any way from the value of the authors' work. While he feels that knowledge of the variation in c_d with velocity gradient is important, he also feels that knowledge of the variation in c_d with acceleration of flow is of at least equal importance. In all probability both velocity gradient and flow acceleration are of significance, and must be

considered together. While several papers[8,9,10,11] have been published concerning the effect of accelerations on the resistance of cylinders, this is the first paper, to the writer's knowledge, that is concerned with the effect of velocity gradients on resistance. For this reason it is a most welcome contribution.

Although neither the aforementioned papers nor the authors' paper give information that can be used directly by designers, they do illustrate the importance of velocity gradients and flow acceleration on drag forces. The writer wishes to congratulate the authors for making the results of their investigation available to the profession and to express the hope that they have thus stimulated efforts to determine reliable design criteria.

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8 "Fluid Resistance to Cylinders in Accelerated Motion," by S. Russell Keim, Proceedings, ASCE, Vol. 82, December, 1956, p. 1113.

9 "Ocean Wave Forces on Circular Cylindrical Piles," by R. L. Wiegel, K. E. Beebe, and James Moon, Proceedings, ASCE, Vol. 83, April, 1957, p. 1199.

10 "Water Forces on Accelerated Cylinders," by A. D. K. Laird, C. A. Johnson, and R. W. Walker, Proceedings, ASCE, Vol. 85, March, 1959, p. 99.

11 "Water Eddy Forces on Oscillating Cylinders," by Alan D. K. Laird, Charles A. Johnson, and Robert W. Walker, Proceedings ASCE, Vol. 86, November, 1960 p. 43.

PREDICTING STORM RUNOFF ON SMALL EXPERIMENTAL WATERSHED[a]

Discussion by Jaime Amorocho

JAIME AMOROCHO,[4] F. ASCE.—The author has presented interesting data that show the inherent limitations of the unit hydrograph approach as a method of predicting the outflow response of a watershed to individual inflow episodes. However, some of the interpretations given in the paper to explain the apparently irregular behavior of small watersheds in this regard deserve further scrutiny.

In accordance with the unit hydrograph principles, the ordinates of any storm hydrograph are directly proportional to the ordinates of a unit hydrograph of the same basic duration. The factor of proportionality is the effective intensity of the storm expressed as a multiple of the unit rainfall excess. This may be said more formally by stating that the unit response to any net inflow episode is an invariant function of the physical characteristics of the catchment. The term "net inflow" is meant to signify the portion of the gross inflow that becomes surface runoff. The principle of superposition implicit in this general assumption is extended to any time sequence of inflow rates and precludes the possibility of using different unit response functions (unit hydrographs) as the inflow rate changes.

In very few if any of the field studies performed to date can it be claimed that the unit hydrograph theory has been tested strictly. This is because in practically no instance has it been possible in nature to ascertain with any degree of assurance that a storm pattern that caused a flood hydrograph actually measured, was proportional in all its elements to the storm used to derive the unit hydrograph under analysis. Other factors, such as the uncertainty in the determination of the infiltration rates used for the construction of the unit hydrograph and for the predicted flood hydrograph contribute very importantly to the weakness of such tests. Hence, the fact that the approximate outlines of recorded flood hydrographs have been reproduced in a number of instances on the basis of a unit graph previously derived may appear to be largely fortuitous. For every instance of approximate agreement there have been very many instances of gross discrepancy. It is a well-known fact that the unit graphs derived from large floods usually differ from those derived from minor floods.

The watersheds investigated by the author are, by virtue of their small areal extent, particularly well suited for a critical check of the application of the unit hydrograph concepts. As a direct result of the reduced areas that make the occurrence of very complex geometric patterns of rainfall with steep, non-stationary intensity gradients less likely within the catchment boundaries, the conditions required for close overall proportionality of inflow can be expected to become more possible. Larger watersheds normally

[a] August, 1960 by Neal E. Minshall (Proc. Paper 2577).

[4] Acting Asst. Prof., Univ. of California, Berkeley, Calif.

produce an effect of attenuation of the rainfall variability that occurs at different locations, provided no major discontinuities exist in the storm pattern over the catchment. In these cases the outflow hydrograph becomes rather insensitive to inflow intensity changes recorded at individual precipitation stations, and its shape tends to reflect the catchment characteristics only. However, because major rainfall discontinuities are more likely in extensive watersheds, the conditions conducive to proportionality of inflow are, in effect, rare.

The author has derived unit hydrographs on the basis of floods produced by short storms of different, "reasonably uniform" intensities. The hydrographs shown in Fig. 6 were presumably reduced to the same unit rainfall excess duration in order to render them consistent with each other. This is not indicated in the paper, but because the observed rainfall excess durations, as reported in Table 3 are approximately the same, no large error is to be expected if the conversion was omitted. The striking fact is that the hydrographs represent distinctly different outflow responses.

A basic premise of the unit hydrograph procedure is that the relationships between storage and outflow are linear.[5] This results from the requirements for a linear solution with respect to the net inflow rate of the storage equation for a hydrologic unit. To illustrate, consider a basin receiving a constant rate of inflow.

The storage equation can be stated as

$$I = Q + \frac{dS}{dt} \qquad (2)$$

in which I is the inflow rate, Q represents the outflow rate and dS/dt denotes the rate of change in storage. This equation has an immediate linear solution in I provided S and Q are related linearly. For if we can write

$$S = K\,Q \qquad (3)$$

K being a constant, then Eq. 2 becomes

$$I = Q + K\frac{dQ}{dt} \qquad (4)$$

that integrates to

$$Q = I\left(1 - e^{-\alpha t}\right) \qquad (5)$$

In this equation, the coefficient $\alpha = 1/K$ in the exponent is a characteristic parameter of the basin. The constant K has the dimension of time and represents the lag due to a "linear reservoir" as defined by Eq. 3. The equation represents the rising limb of the instantaneous unit hydrograph. The equation of the recession limb follows from integration of Eq. 4 for $I = 0$ at $t = t_p$:

$$Q = Q_p\, e^{-\alpha\left(t - t_p\right)} \qquad (6)$$

The subindex refers to the values at the moment of cessation of the inflow. Now, because by Eq. 5 the discharge at peak is

$$Q_p = I\left(1 - e^{-\alpha t_p}\right) \qquad (7)$$

Eq. 6 can be written

$$Q = I\left(e^{-\alpha\left(t - t_p\right)} - e^{-\alpha t}\right) \qquad (8)$$

It is seen that Eqs. 5 and 8 are linear functions of I, that is the basic premise of the unit hydrograph method. J. C. I. Dooge,[5] M. ASCE, has made

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[5] Journal of Geophysical Research, J. C. I. Dooge, February, 1959.

an exhaustive analysis of the unit hydrograph theory in which variability of the inflow rate is accounted for.

In natural watersheds, the assumptions of linear reservoirs and linear translation processes can not be taken for granted. The terms S and Q are normally related by non-linear functions which contain additional variables such as friction and velocity gradients. Introducing also the space variability of the inflow rate, the storage equation has the general form

$$I(t, x, y) = Q + \frac{d}{dt} f\left(Q, S_f, x, y, z\right) \qquad (9)$$

It can be easily surmised that this equation will, in general, not have a linear solution in I.

Practically all cases in nature are of this complexity; therefore, it may be stated quite generally that the classical unit hydrograph concept can almost never be applied strictly. It should be emphasized that the preceding conclusion governs not only the proportionality between the ordinates of the runoff hydrograph and the unit hydrograph but the additive superposition of the effects of variable inflow rates as well.

Some rather interesting conslusions may be derived from Figs. 5 and 6, that illustrate the previous statements. The derivation of unit hydrographs from storms of varying intensities give rise to inaccuracies that depend on subjective factors that inevitably creep into the process of separation of the different component runoff episodes. Inaccuracies also result, as mentioned previously, from the determination of the infiltration rates, that, in the present case, was made on the basis of a completely empirical criterion. Nevertheless, some measure of verification of the latter was afforded by the observed actual runoff, and the clear trends apparent in the alignment of points plotted in Fig. 5 appear reliable. From these trends it may be concluded (1) that the shape of the unit graph is a non-linear function of the inflow rate; and (2) that depending on whether the unit graph corresponds to an initial period of inflow or to inflows that occur after some runoff created by previous rain is taking place, the response is different although the intensity may be the same. This may be interpreted as a "field proof" of the inapplicability of the principle of superposition in this case.

It is obvious that the exact relationships suggested by the data of Fig. 5 apply only to the particular watershed described in the paper. Other relationships will exist in different catchments, but as long as their empirical determination is possible, the type of analysis that follows should have general application. The only requirement is that the overall geometry of the inflow pattern should not vary greatly within the basin boundaries for different rain intensities.

It is noted that the lines of the upper and lower graphs of Fig. 5 can be cross-plotted to define a direct relationship between the hydrograph peak and the time to peak. Scaling values from the figure, it is found that the following expressions result:

$$\text{For unit response early in storm: } q_{max} = 15.25\, t_p^{-0.625} \qquad (10)$$

$$\text{For unit response late in storm: } q_{max} = 28.55\, t_p^{-0.793} \qquad (11)$$

in which q_{max} is the maximum ordinate of the unit response and t_p is the time to peak. These equations represent the loci of the peaks of hydrographs

such as those shown in Fig. 6. These hydrographs may be considered as families of curves that must satisfy the following conditions:

1. To have maxima passing through the lines described by equations such as Eqs. 10 or 11.
2. To encompass an area of unity between each curve and the time axis (by the definition of the unit hydrograph).
3. To have 0 value at time equal zero and to be asymptotic to the time axis.
4. To be continuous and have a positive second derivative near the origin and a first derivative equal to zero at the origin.

These conditions are met for example by some asymmetric frequency distributions. It may be noted by reference to Fig. 6 that the curves tend to become symmetrical as the time to peak increases. We might, therefore, choose a position in which this symmetry is closely approximated and fit therein a normal frequency distribution to represent the unit response. The ordinate at time zero would have a finite value, but the error thus introduced in meeting conditions 2 and 4 would probably be small. This basic function could then be skewed and peaked simultaneously by introducing appropriate parameters in such a way that Eqs. 10 or 11 would be satisfied for each value of t_p. A similar approach could conceivably be used with other functions, in order to arrive at some empirical expression for the unit response as a function of the inflow intensity.

It is desirable, however, to use a function that is meaningful in terms of a theoretical analysis of watersheds. J. E. Nash[6] and Dooge[5] derived independently the general expression of the outflow function for a catchment with n linear reservoirs and a corresponding number of linear channels. This expression has the form of a Poisson distribution and represents the unit hydrograph of a linear catchment of this type:

$$q(t) = \frac{\alpha (\alpha\, t)^{n-1}\, e^{-\alpha t}}{\Gamma(n)} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

in which $\alpha = \frac{1}{K}$ has the same meaning as in Eq. 5.

The values of α and n can be computed from observed outflow hydrographs by the method of moments as described by Nash. A simple procedure, that assures that the maximum value of q satisfies Eqs. 10 or 11 exactly, although the fit may be less precise elsewhere, consists of finding the solutions for α and n from the system formed by Eq. 12 and the relationships

$$q_{max} = \delta\, I^{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

and

$$t_p = \delta_1\, I^{-\lambda_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

obtained from the data of Fig. 5. The effect of the imperfect fit of the equation to the unit response derived from the data is of little practical consequence as will be discussed later.

For q maximum, the first derivative of Eq. 12 must be zero:

$$q'_{t_p} = \frac{\alpha^n}{\Gamma(n)} \left[(n-1)\, t_p^{\,n-2}\, e^{-\alpha\, t_p} - \alpha\, t_p^{\,n-1}\, e^{-\alpha\, t_p} \right] \ldots \quad (15a)$$

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[6] Proceedings, J. E. Nash, Genl. Assembly of Toronto, Internatl. Assn. of Scientific Hydrology, September, 1957.

$$q'_{t_p} = \frac{\alpha^n}{\Gamma(n)} e^{-\alpha t_p} \left[(n-1) t_p^{n-2} - \alpha t_p^{n-1}\right] = 0 \qquad (15b)$$

Hence

$$(n-1) = \alpha t_p \qquad (16)$$

and combining Eqs. 16, 12, 13 and 14 yields

$$q_{max} = \frac{(n-1)^n}{\Gamma(n)} \frac{e^{-(n-1)}}{S_1 I^{-\lambda_1}} = \delta I^{\lambda} \qquad (17)$$

or

$$\frac{(n-1)^n e^{-(n-1)}}{\Gamma(n)} = \delta \delta_1 I^{\lambda - \lambda_1} \qquad (18)$$

An explicit expression for n in terms of the experimental parameters δ, δ_1, λ and λ_1 can be obtained from Stirling's approximate formula for $\Gamma(n)$:

$$\Gamma(n) \sim (2\pi)^{1/2} (n-1)^{(n-1)+1/2} \exp.\left[-(n-1) + \frac{1}{12(n-1)}\right] \qquad (19)$$

that, when substituted in Eq. 18, yields,

$$(n-1) e^{\frac{1}{6(n-1)}} = 2\pi \delta^2 \delta_1^2 I^{2(\lambda - \lambda_1)} \qquad (20)$$

For $2 < n < 9$ it is found that

$$(n-1) e^{\frac{1}{6(n-1)}} \approx n - 1.17 \qquad (21)$$

Therefore, Eq. 20 becomes

$$n = 2\pi \delta^2 \delta_1^2 I^{2(\lambda - \lambda_1)} + 1.17 \qquad (22)$$

Likewise, from Eqs. 13 and 14 we get

$$\alpha = 2\pi \delta^2 \delta_1 I^{2\lambda - \lambda_1} + \frac{0.17}{\delta_1} I^{\lambda_1} \qquad (23)$$

From the data presented in the paper we find that $\delta = 1.170$; $\delta_1 = 0.483$; $\lambda = 0.710$; and $\lambda_1 = 0.547$ as a result of which

$$n = 2.0090 I^{0.326} + 1.17 \qquad (24)$$

and

$$\alpha = 4.1569 I^{0.873} + 0.3518 I^{0.547} \qquad (25)$$

Fig. 11 shows a comparison between the hydrographs presented by the author in Fig. 6 and the curves computed by means of Eqs. 12, 24 and 25. It is noted that the fit becomes increasingly better for higher values of the inflow intensity. Similar fits computed by the method of moments were found to be slightly better for the curves corresponding to low intensities but became increasingly difficult at higher intensities. The precision of the fit is within the order of magnitude of the error to be expected from the field data, as may be surmised from the scatter of the points shown in Fig. 7. In general, slightly higher runoff volumes would correspond to the areas below the fitted curves during the first hour than to the areas below the curves shown in Fig. 6.

Eqs. 20 and 23 suggest that the hydrologic behavior of a watershed may be characterized by functions that define the number of equal theoretical linear reservoirs and the theoretical lag time of one linear reservoir in terms of the inflow rate. The substitution of these expressions in Eq. 12 gives an equation that can be considered as the approximate solution of Eq. 9 when I does not vary in time. An interesting implication of these equations is that a watershed may be regarded as equivalent to a linear catchment only when the inflow is constant. As soon as the inflow rate changes, the number of theoretical reservoirs becomes larger or smaller and the response varies accordingly. Therefore, neither n nor α may be considered as invariant characteristic

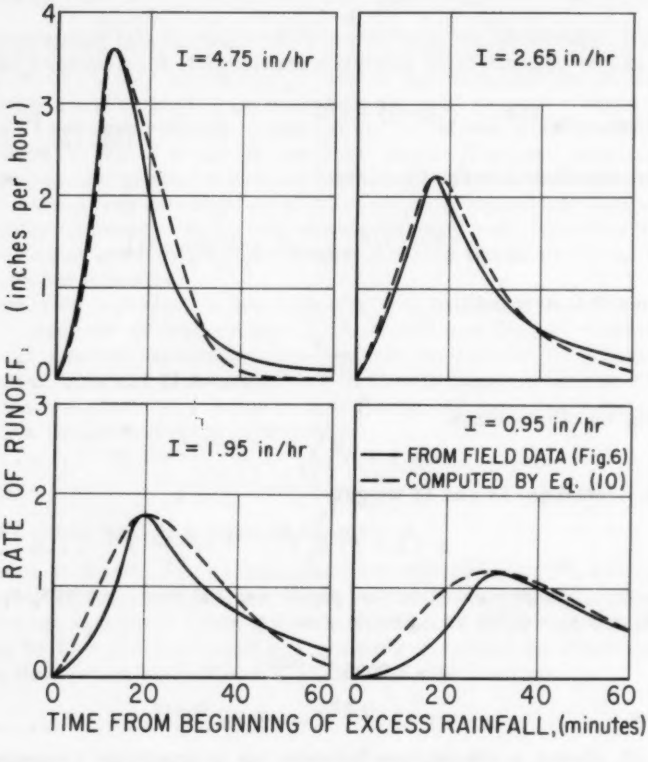

FIG. 11.—COMPARISON OF UNIT RESPONSE CURVES FOR DIFFERENT INFLOW INTENSITIES

parameters of a catchment. This conclusion explains the inapplicability of the principle of superposition in this type of hydrologic problem, and may not only be extended to the critical evaluation of the unit hydrograph method but to other related flood routing procedures, such as the Muskingum method, as well.

The preceding analysis suggests a practical procedure for obtaining the response function for small watersheds in which uniform precipitation rates

are recorded. This procedure would consist of first selecting a number of individual runoff hydrographs that can be separated and definitely ascribed to periods of reasonably uniform rainfall excess. The values of peak discharge and time to peak should then be plotted against rainfall intensity and from these relationships, by an analysis similar to that described in this discussion, the expressions for n and α in terms of I could be derived. It should be kept in mind that whatever form these expressions may acquire, their nature is strictly statistical and empirical. Hence, from a practical standpoint there is no object in scrutinizing the physical meaning or the dimensions of the parameters that enter their configuration.

An investigation is in progress (in 1961) at the University of California, Berkeley, Calif., in which these concepts are being studied in model catchments. A full theoretical and experimental treatment of this subject, initiated in 1959, is being carried out under a program of the University's Water Resources Center.

PREDICTING STORM RUNOFF ON SMALL EXPERIMENTAL WATERSHEDS[a]

Discussion by Merwin D. Dougal

MERWIN D. DOUGAL,[7] A. M. ASCE.—Research and experience have well indicated the many variables that must be considered in developing a usable rainfall-runoff relation. Many that have been developed present unique problems in their use in estimating runoff on ungaged watersheds. The author's work extends known techniques both in methods of approach and in providing information on a particular area of the nation.

The writer has studied[8] the problem of a modified infiltration index and the rate of infiltration rather than either total retention or runoff estimates, but it is apparent that either of the latter could be obtained indirectly. The antecedent precipitation index approach was used, and season (or week-of-year) duration of excess rainfall and average intensity of rainfall were the other independent variables given primary consideration. Certainly the Ralston Creek watershed near Iowa City, Iowa is classical in hydrologic research.

The use of the antecedent precipitation index, computed by Eq. 1, appears to be widely applicable. As recommended by Kohler and Linsley,[9] the index should be used in conjunction with season of the year or temperature. The author states, for the various methods and lengths of period tested in computing the index, that the best correlation with retention was a 30-day index plus 20% of the departure from normal of the 30 to 60 day precipitation. It is later stated that an antecedent retention index was used in an attempt for refinement, but the results showed no noticeable improvement over the 30-day index finally selected for use. The recession factor "k" in Eq. 1 was evaluated as 0.95 for the Edwardsville, Illinois watersheds.

The selection of 0.95 for the recession factor, a relatively high value, indicates that previous precipitation affects soil moisture in the watershed to a substantial and somewhat lasting degree, because the API values recess quite slowly. This means that if appreciable precipitation has previously occurred, its effects on soil moisture linger on. Perhaps the author might comment on the effect of the impervious clay-pan layer on the relatively low amount of soil moisture capacity indicated.

The writer initially investigated three values of the recession factor "k" for the Ralston Creek watershed: namely, 0.84, 0.90, and 0.94. In the resulting graphical correlation, the values of 0.84 and 0.90 were about equally capable in enabling first approximation month-of-year curves to be drawn, but

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[8] "A Study of Factors Which Affect Infiltration Rates on the Ralston Creek Watershed," by Merwin D. Dougal. Thesis presented to the Graduate College of the State University of Iowa, at Iowa City, Iowa in February, 1958, in partial fulfillment of the requirements for the degree of Master of Science.

[9] "Predicting the Runoff from Storm Rainfall," by M. A. Kohler and R. K. Linsley, Research Paper No. 34, Dept. of Commerce, Weather Bureau, Washington D. C., September, 1951.

little correlation was evident for the value of 0.94. The mean of the first two values, 0.87 was subsequently selected as most applicable for the Ralston Creek watershed. This watershed is about 3.0 square miles in area, relatively rolling to hilly. The soil is of loessial origin over glacial till with a Clinton silt loam soil surface. The Ralston Creek watershed experiences high infiltration rates.

In regard to the methods and lengths of period available in computing the antecedent precipitation index, the writer would like to add several others. The problem is selecting an initial API value and a proper time interval before a storm in which to recess the initial index. The author does not state what initial value was used in his study. The writer investigated four procedures in the Ralston Creek study:

1. Beginning with a zero value of API 60 days prior to the storm.
2. Beginning with the normal 10-day precipitation for the period prior to 30 days before the storm (first storm on May 1, use as the initial value the 10-day normal precipitation prior to April 1, and begin API computations on April 1).
3. Beginning with the observed average 10-day precipitation for the 30 day period prior to 30 days before the storm (first storm on May 1, use as initial index the 10-day average precipitation of March and begin API computations on April 1).
4. Same procedure as (3), except the 10-day average precipitation is prior to a 3-week period before the storm.

The four procedures, when applied in the Ralston Creek study to periods prior to storms indicated below, yielded the values indicated in Table 5. The results indicate that the initial value need only be reasonably assumed. To illustrate, for the storm of August 4, 1924, the initial value for procedure (2) was 1.58 in., the 10-day normal for June. For procedure (3) the observed 10-day average precipitation for June was 2.47 in. As indicated previously, the respective API values after a 30-day recession period were almost identical. It seems possible to the writer that the use by the author of a different "k" factor in a 30-day index might have given identical results to the 30-day index plus the 20% departure used in the study.

A comment by the author on his method used in computing API values for successive storms within a day would be appreciated. For instance, in Table 1, the same API value is listed for three storms occurring on August 14, 1946. Presumably the API value is higher for successive storms on the same day either by direct addition, or by recessing on an hourly basis.

The writer concluded, in the analysis of API in the Ralston Creek study, that it is simpler to carry the API computations forward continuously from the first storm in a particular year to the last storm. Less computations are involved than if separate API computations are made for each storm. In addition, this continuous method would be used in flood forecasting techniques.

The writer agrees that a large volume of data is needed to develop graphical correlations in a hydrologic analysis. Over 258 storms in a 1927 to 1956 record were used by the writer and all were utilized to advantage. However, about 100 of the storms were selected as having "ideal" storm characteristics and were used in initial correlation efforts of API and season, month-of-year, or week-of-year.

The author indicates the variation in season to season that can be expected. Because the final grouping refined some periods to 30-day periods, it is of

interest to inquire of the author's method for allowing for the seasonal variation in plotting the data. The writer attempted to use temperature as a parameter (departure from the mean weekly temperature for the week prior to the storms under consideration). It was concluded that if all other variables were about constant, temperature effects were of importance, but normally were too obscured by other variables to be of value.

The writer believes that the abscissa in Fig. 3 could be more aptly termed "Duration of excess rainfall, in hours" because the plotted points indicate a single total storm event and not the variation in retention during a storm. The author also assumed that infiltration capacity is independent of rainfall intensity. This is a perplexing problem to analyze because in cultivated areas the intensity may effect surface cover conditions, especially in the spring months. The comparison of estimated and observed runoff indicated in Fig. 4 shows best results for runoffs above one inch, and it is above this that the

TABLE 5[a]

Date of Storm	Procedure			
	1	2	3	4
August 4, 1924	--	0.95*	0.97*	--
March 14, 1934	0.14	0.14	0.14	0.16
May 3, 1946	0.83	0.81	0.76	0.81
May 28, 1947	0.82	0.80	0.82	0.80
*k = 0.87				

[a] API (k = 0.90)

engineer is normally concerned in estimating storm runoff. For use in estimating past yields, individual estimates might be in error, but an acceptable average should be obtained over a period of days or weeks.

The writer, in the Ralston Creek study, was unable to correlate certain storms that appeared on further analysis to possess a common characteristic, that is, a fair amount of intense rainfall had occurred within a few hours (24 hr period maximum) before the beginning of the storm being scrutinized. This was finally attributed to an additional variable, namely, the variation or difference between a rising and falling API at the time a storm occurs. It is easily reasoned that there is considerable difference in soil surface conditions whether the API is falling or rising to the index existing immediately before occurrence of the storm rainfall. To illustrate, assume an API value of 2.0 in. exists immediately prior to each of two hypothetical storms. For one storm the API of 2.0 in. has resulted from the log arithmic recession of a 5 in. to 6 in. rainfall a week or more before the storm in question. For the second hypothetical storm the API of 2.0 in. resulted primarily from a 2 in. intense rainfall a few hours before the storm in question. For these two hypothetical storms, all other conditions being the same, the obvious difference in soil moisture at the surface and in soil surface conditions will cause infiltration values for the two storms to be much different. The writer introduced shift-curves to account for this phenomena that enable, in effect, a shift to be made to a higher API value when a rising API situation was encountered, thus giving a lower infiltration rate.

The author has accomplished an admirable evaluation of hydrographs on small watersheds. The variation of the peak rate of runoff with both rainfall intensity and pattern of rainfall intensity is of interest. In regard to the variation with intensity, is it possible that the high intensity causes surface depths to reach a point at which not only is the flow velocity increased appreciably, but surface vegetation is altered; that is, flattened, disrupted, and so on? The author's results, as indicated in Table 4, reveal a good correlation, with most values for large amounts of storm rainfall being within 10%.

The writer would like to inquire if the author compared his synthetic hydrographs with those that would be computed by present techniques used by the Soil Conservation Service and developed by the Agricultural Research Service, U. S. Department of Agriculture. An examination of Fig. 9 and Fig. 5 indicates that the former is based on the occurrence of high intensity rainfall late in a storm. Perhaps the author can state which of the two effects, early or late, is more commonly experienced in nature.

The results of the author's study are stimulating. Additional research efforts in other parts of the nation should be encouraged to aid the hydrologist and the engineer in his design capacity. A recompilation and analysis of Ralston Creek data has been completed at the University of Iowa, Iowa City, Iowa, and is to be published (1961) in cooperation with the Iowa Highway Research Board and other agencies. This report should also add to the field of knowledge concerning small watersheds.

ROBERT L. McFALL[10] and BEN A. JONES, JR.[11]—The validity of the proposed method was tested with data from a central Illinois watershed maintained by the Agricultural Experiment Station of the University of Illinois. The watershed is located near Monticello, approximately 120 miles northeast of Edwardsville, Illinois. From the limited data available, two storms over the 82-acre watershed were selected for comparison.

TABLE 6.—STUDY OF CENTRAL ILLINOIS WATERSHED

Date of Storm	30-day API	Peak Runoff Rates, in inches per hour		Total Runoff, in inches		Excess precipitation, in inches
		observed	computed	observed	computed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
June 27, 1951	0.78	0.51	0.58	0.56	0.50	1.61
July 9, 1951	1.48	0.68	0.83	0.82	0.91	2.11

The watershed is predominantly silt loam soil and has slopes that are generally less than 4%, typical of the dark-colored, moderately permeable gently sloping prairie soils of east central Illinois.

The time retention curves proposed by Minshall were used in computing runoff rates. The 30-day antecedent precipitation index (API) was computed using the same formula used by Mr. Minshall except the K value was changed to 0.90. (There is some question in the minds of the writers as to whether a 30-day index should be used, but data are lacking to test this parameter.)

10 Instr., Univ. of Illinois, Urbana, Ill.
11 Assoc. Prof., Univ. of Illinois, Urbana, Ill.

The results of the study are shown in Table 6 and one of the hydrographs is presented in Fig. 12. Table 6 shows that the observed peak runoff from both storms was less than the computed peak runoff. But the observed total runoff was higher than the computed value for one storm and the reverse was true for the other storm. A study of the hydrograph shows that a change in the shape of the portion of the hydrograph that falls below the 15% runoff rate could change the volume of runoff considerably. Thus, the total runoff is a weaker comparison tool than the peak discharge rate and the time-rate change of the hydrograph.

Because the time retention curves presented by the authors were computed from data from clay-pan prairie soils, the use of a soil retention factor, in addition to the seasonal parameters, is suggested. Although the time retention curves provide a means for determining the runoff, their use requires six

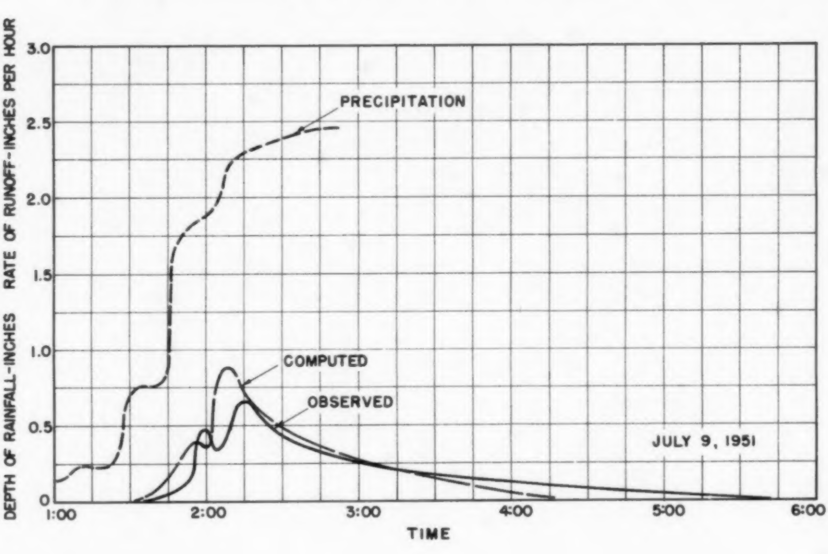

FIG. 12

families of curves for each variation in soil retention characteristics. The data and retention curves presented were studied, and it appears feasible to construct a coaxial correlation that would contain, in addition to the seasonal parameters, information about the soil characteristics. This proposed method will greatly facilitate the determination of precipitation retention when working with widely varied soil conditions. More data and exploration are needed to confirm this opinion.

The tests conducted indicate that the method proposed by Minshall for synthesizing hydrographs will lead to peak runoff values that are higher than observed flows. It is impossible with the limited data available to predict whether total runoff will be higher or lower than observed values. In these cases the difference in values does not exceed 15%. These are good results considering

the limited knowledge about some of the factors. The writers believe Minshall's method shows promise and its applicability to a number of soil, cover, and climatic conditions should be investigated by other workers.

PROCEEDINGS PAPERS

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DECEMBER: [illegible] 2670(EM6), [illegible] 2672(EM6), [illegible] (EM6), [illegible] 2679(ST12), [illegible] 2680(ST12), 2681(SM6), [illegible] 2684(SM6), [illegible] 2689(EM6), [illegible] (ST12), [illegible] 2697(ST12)[c].

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c. Discussion of [illegible] contained [illegible]